NONLINEAR AC PROBE MEASUREMENTS
IN A COLLISIONLESS PLASMA

by

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CERTIFICATE OF APPROVAL

MASTER'S THESIS

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ABSTRACT

An investigation of the a. c. potential variation in the vicinity of a planar electrode in a collisionless argon plasma was made at frequencies much lower than the plasma frequency. Nonlinearities were observed which were studied as a function of bias voltage applied to the planar electrode and distance from the planar electrode. The primary feature of these nonlinearities was a minimum in the a. c. potential which changed as a function of distance and bias voltage. An investigation of the point at which the minimum in a. c. potential occured as a function of negative bias voltage on the planar electrode was made. The overall response of the system was compared to an a. c. equivalent circuit in which the dominant changes were due to changes in the capacity of the sheath formed near to the planar electrode. This model did not predict the observed minimum in a. c. response, and suggestions were made as to the mechanism responsible.

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I. INTRODUCTION

Considerable work has been done in laboratory plasma experiments at frequencies well below the plasma frequency. Experiments by Takayama, Ikejami, and Miyasaki [1960] with resonance probes biased at a DC level, which deal with frequencies near the plasma frequency, have yielded means of measuring number densities, electron temperatures, etc. Lower frequencies have been used to study the characteristics of the positive sheath which surrounds a probe when placed in a plasma, and to evaluate the plasma parameters at these frequencies. Several models of the sheath and the sheath impedance have been presented. The sheath criteria first derived by Bohm [1949] has been used by Crawford and Grard [1966] to evaluate the sheath impedance.

A laboratory experiment was performed with the intention of studying the potential near a plane electrode placed in an argon plasma with an AC voltage of an amplitude of 10 volts rms, at frequencies from 1 to 15 kHz. This probe was biased with a constant potential relative to the plasma. It was observed that for certain separations of the transmitting probe and the probe used to measure the potential in the plasma, the AC signal received became highly nonlinear. This nonlinearity was observed to be a function of both the DC biasing voltage, the frequency of the AC signal, the probe separation, and to a certain extent the amplitude of the AC signal. A study was undertaken to investigate the nature of the nonlinearities and suggestions made to explain their production.

II. DESCRIPTION OF EQUIPMENT

The plasma is formed in a glass bell jar, 12 inches in diameter and 12 inches in height. A schematic diagram of the apparatus is shown in Figure 1, and the AC equivalent circuit is shown in Figure 2. Figure 3 shows a photograph of the apparatus with the bell jar in the open position and Figure 4 with the bell jar in the closed position.

The plasma is formed by ionization of argon gas in the chamber. Electrons are accelerated by the potential difference between a hot tungsten filament and a wire screen anode. The anode is in the shape of a rectangular solid two and one-half inches wide, two inches high, and two and one-half inches in depth. It is constructed of stainless steel screen and has one side open to admit the filament. The filament is a coiled tungsten wire which is biased negatively with respect to the anode which is at ground potential. The bias voltage must be greater than the ionization potential of argon in order for the plasma to be formed. Typical operating voltages are 100 to 200 volts.

The chamber is evacuated by a mercury vapor diffusion pump which continually pumps on the chamber as argon gas is leaked into the chamber through a calibrated leak valve. The pressure of the chamber can be kept approximately constant which produces a plasma homogeneous in the region near the center of the bell jar. Typical operating pressures are from 10^{-3} to 10^{-4} mm Hg.

The bell jar is mounted on a stainless steel plate which has openings for the vacuum pump, a pressure gauge, and the leak valve. Provisions are made for electrical connections through the plate and for mechanical manipulations of the probes while the chamber is in operation. The plate is taken as ground potential for all voltages used in the experiment.

Two probes are mounted on insulating supports which are connected to a chain. The chain is used to change the separation of the two probes. The transmitting probe is a copper plate cut in the shape of a square with sides one inch long. The plate is connected to coaxial cable which leads to the outside of the chamber.

The receiving probe is cylindrical to avoid perturbing the plasma as much as possible. The receiving probe is insulated from the plasma with the exception of the horizontal end section which is 0.032 inches in diameter and 0.36 inches in length. The receiving probe is connected by coaxial cable to an amplifier with an input impedance of 10^8 ohms and a gain of two. This amplifier is connected to a high pass filter which rejects signals below a frequency of 135 Hz. The purpose of this filter is to eliminate a large amount of noise at 60 Hz caused by the AC power lines. The high pass filter is followed by an amplifier with a gain of 100 which is connected to the input of an oscilloscope for observation of the received signal.

The transmitting probe is connected to an oscillator which can be biased at a constant potential with respect to the stainless steel plate. A 100 ohm resistor in series with the oscillator provides an indication of the current delivered by the oscillator by measuring the AC voltage drop across the resistor.

The potential of the transmitting probe can be measured directly with a DC voltmeter and the average current drawn by the probe can be computed from the voltage reading of the voltmeter and the voltage across the DC supply.

The number density and the electron temperature of the plasma can be computed by use of a Langmuir probe which may be moved inside the chamber to prevent interfering with the signal between the transmitting probe and the receiving probe. The Langmuir probe is a cylinder with a diameter of $0.050 \pm .002$ inches and a length of $0.245 \pm .002$ inches. A DC power supply and a microammeter are used to obtain the Langmuir probe characteristics.

This apparatus has the advantage of producing a homogeneous plasma near the center of the jar for a long period of time, and in a large volume. Mechanical adjustments may be carried out easily while the chamber is in operation. The number density and electron temperature may be changed by a change in the operating parameters: the filament temperature, the anode voltage, the pressure, and the transparency of the anode screen. The apparatus has the disadvantage of lacking control in maintaining a constant pressure over a long period of time. Small variations in pressure cause noticable fluctuations in number density, which in turn may effect an experiment in progress.

III. MEASUREMENT OF PLASMA PARAMETERS

In order to measure the parameters of the plasma, number density, electron temperature, etc., the conventional Langmuir probe techniques were used. This method can be found as done by Langmuir and Mott-Smith [1924]. Discussions of probe technique can also be found in numerous other papers, for example, Kagan and Perel' [1964] and Chen [1965].

The Langmuir probe curve is as shown in Figure 5. When the probe is very negatively charged it collects ions and repells all electrons. The curve is not completely flat because the ion current is increased slightly due to the attraction of some ions which would not have reached the probe at a slightly higher potential. For very large positive potentials the probe draws saturation electron current and no ion current. Again the curve rises slightly, but is fairly flat. The transition region between large positive and large negative potentials is where the plasma parameters are measured. It may be shown, assuming the velocity distribution of the electrons to be a Maxwellian distribution, and ignoring the ion current compared to the much larger electron current, that this part of the curve may be described by the relation

$$I = eAn_o\left(\frac{kT_e}{2\pi m}\right)^{1/2}\exp(-eV_p/kT_e) \tag{1}$$

where A is the area of the collecting surface,

n_o is the number density of the plasma,

T_e is the electron temperature,

m is the mass of the electron, and

V_p is the magnitude of the probe potential.

Taking the logarithm of both sides of the above equation we have

$$\ln(I/I_s) = \frac{-e}{kT_e} V_p \tag{2}$$

where $I_s = eAn_o(kT_e/2\pi m)^{1/2}$.

Thus if we plot the natural logarithm of the current, I, drawn by the probe as a function of the probe voltage, V_p, we obtain a linear plot in the transition region with slope e/kT_e. This determines the electron temperature. The number density may be determined by extrapolating the linear plot of the transition curve and the linear plot of the electron saturation curve to their intersection. This point is where the plasma and the probe are at the same potential and V_p is equal to zero. Here the current collected by the probe is the randomn electron current I_s. Evaluation of I_s determines the number density of the plasma.

The electron saturation region can be shown to follow the relation

$$I = \frac{eA}{\pi} n_o \left(\frac{2kT_e}{m}\right)^{1/2} \left(\frac{eV_p}{kT_e} + 1\right)^{1/2} \tag{3}$$

thus if I^2 is plotted as a function of V_p there is a linear region of slope S,

$$S = \frac{2e^3 A^2}{m\pi^2} n_o^{\ 2} \tag{4}$$

which determines the number density of the plasma independently of the electron temperature.

Using these techniques the measured number density for the plasma was found. The number density as determined by the conventional Langmuir probe technique was 1.39×10^{13} m^{-3}, and the electron temperature was found to be 0.35 electron volts or 405°K. Using the second technique the number density was measured at $7.95 \times 10^{12} m^{-3}$. These values agree to within a factor of about two. The rather large discrepency appears to come from the graphical construction needed to find the value of I_o in the first method used. A small shift in the slope of the linear part of the transition region produces a small change in the electron temperature, but a large change in the value of I_o, and hence, in the measured number density. The change in the slope due to the uncertainties of the data points can change the value obtained for the number density by a factor of one and a half to two. The first method used assumes that the diameter of the sheath is nearly the same as the diameter of the probe. The second assumes that it is much larger than the probe diameter.

Using a number density of $1.0 \times 10^{13} m^{-3}$ and an electron temperature of 0.35 electron volts, the following set of parameters for the plasma may be derived,

$$\omega_p = 1.8 \times 10^8 \text{ Hz}$$
$$\lambda_D = 1.4 \times 10^{-3} \text{ meters}$$
$$t_c = 1.90 \times 10^{-5} \text{ seconds}$$

where ω_p is the plasma frequency, λ_D is the Debye length, and t_c the self collision time of the electrons.

The neutral argon atoms are ionized by collisions with the electrons given off by the filament, and the plasma drifts out into the chamber until an equilibrium is attained. The plasma is homogeneous near the center of the chamber as can be ascertained by direct measurement with the Langmuir probe. The cross section for an electron-argon collision at 0.35 electron volts is $0.3 \times 10^{-20} m^2$, while at 10 electron volts the cross section is $14 \times 10^{-20} m^2$, as measured by Barbiere [1951], Kivel [1959], Frost [1960], quoted by Uman [1964]. The mean path length for a collision at 0.35 electron volts is 27.6 meters, which decreases to 0.59 meters at an electron energy of 10 electron volts. This minimum in the cross section for electron-argon collisions makes path lengths for the higher energy ionizing electrons short enough to produce a sufficient number of collisions to create the plasma. The secondary thermal electrons, however, have mean free path lengths many times the dimensions of the plasma chamber, and thus the plasma can be considered collisionless with regard to the thermal electrons. The neutral number density can be estimated at $2.2 \times 10^{13} cm^{-3}$ assuming the neutral gas is at room temperature and the measured pressure is at 6.9×10^{-4} mm Hg.

IV. LABORATORY AC PROBE MEASUREMENTS

The following experiment was carried out with the plasma parameters as previously described. A sinusoidal potential biased at a constant DC level was applied to the transmitting plate. At the receiving probe a highly non-linear response was noted for certain values of bias voltages and probe separations.

Figure 6 shows the response observed due to a variation in the probe separation. The probe separation, accurate to the nearest millimeter, is noted by X; the measured DC bias voltage on the transmitting plate by V. The pictures are in sequence from the largest probe separation to the smallest. The changes in the values of V are consistant with random variations in time caused by small changes in the pressure in the chamber. The value of V for X = 10.0 cm is quite different than the others because the transmitting plate is near the wall at this position.

As can be seen in Figure 6, the signal received is essentially sinusoidal at large probe separations. As the probes are moved together the signal strength increases until the signal becomes distorted. At one point the signal has a large component of the second harmonic at 10 kHz. As the probes are moved together the distortion disappears and a sine wave is regained.

Figure 7 shows the received signal as the separation of the probes is fixed at 4.0 cm and the bias voltage on the transmitting probe is changed. It can be seen that at large positive plate voltages the signal is nonlinear. The signal form remains qualitatively unchanged until the bias voltage nears zero. At this time the signal amplitude is much smaller than it was for larger plate bias voltages. The wave form also begins to change shape from the wave form at positive bias voltages. The signal amplitude becomes very small and the same type of distortion is noticed as was in the previous case. The fundamental frequency component has a very low amplitude relative to the second harmonic and the signal has essentially twice the frequency of the imput signal. As the plate voltage becomes more and more negative, the distortion eventually disappears and the sinusoidal wave form reappears.

Figures 8-12 show a quantitative break down of the first three frequency components present in the non-linear response. Figure 13 shows a detail of the portion of Figure 11 at which the plate draws positive current. Five different probe separations are shown. The amplitudes of the frequency components are plotted as a function of the positive current drawn from the plasma by the transmitting plate. The following characteristics may be noted for all five probe separations.

The overall response as a function of plate current is as follows. For large negative current drawn by the plate, that is, the plate at a large positive bias voltage and drawing electrons, the received signal has its largest value. The amplitudes of all harmonics drop with increasing plate current until at positive currents (large negative bias voltages) the amplitudes of the harmonics have values much smaller than their values at large negative currents.

It may be noted that for each of the five different probe separations there is a definite minimum in the amplitude of the fundamental frequency component at small positive plate currents. The probe separation at which this minimum occurs increases as the bias voltage on the transmitting plate becomes more and more negative. There is also a tendency for the amplitude of the second harmonic to fall off at the largest negative currents drawn by the probe.

Figure 14 shows a plot of the probe separation at which the minimum occurs as a function of the negative bias voltage on the transmitting probe. The plot is linear on a log-log plot, indicating a possible power law connecting the probe separation and the bias voltage. Because of the flatness of the minimum point with regard to changes in bias voltage, fluctuations in the plasma occuring with changes in the pressure in the chamber, and the low signal level at the minimum point, it is very difficult to obtain an accurate plot. The current collected by the transmitting plate was, in each case, positive. This indicates that the potential varies as a function of distance in such a manner as to collect more ions than electrons. The current collected by the probe seemed to increase slightly as the probe separation increased, averaging +10.54 μamps, with the lowest value +7.98 μamps and the highest +12.18 μamps.

V. THEORY OF SHEATH FORMATION AND IMPEDANCE

In order to understand the overall change in amplitude it is necessary to derive an expression for the impedance of the sheath formed at the transmitting plate. The sheath is formed because of the difference in velocity between the ions and the electrons in the plasma. Because of this difference the probe collects more electrons than ions during a transient period, takes on charge, and eventually attains an equilibrium. Due to the net negative charge on the probe the electrons with lower velocities are reflected from the probe and a net positive charge density forms a layer near the surface of the probe. This layer of positive charge is called the sheath. The thickness of the sheath can be estimated by considering the probe and the approximate outer boundary of the sheath to be plane electrodes of different potential. Ions are emitted at the boundary between the sheath and the plasma and are collected at the surface of the probe which is considered to be a perfect absorber. Since all but the most energetic electrons are deflected by the negative potential of the probe, the electron velocity distribution is considered to be a Maxwellian distribution at the edge of the sheath boundary.

This is not strictly true since the most energetic electrons are collected by the probe and are missing from the distribution. The number of electrons missing is small, however, and the actual distribution may be approximated by a Maxwellian distribution.

Using the Child-Langmuir law we can write that the ion current collected by the probe is given by

$$I_p = \frac{4}{9}\varepsilon_o A\left(\frac{2e}{Mi}\right)^{1/2}\frac{(-V_p)^{3/2}}{d^2} \tag{5}$$

where d is the thickness of the sheath,

A is the area of the probe,

M_i is the mass of the ions, and

V_p is the potential of the probe relative to the plasma.

The Child-Langmuir law, however, assumes that the ions leave the plasma with zero initial velocity which cannot be the case. Bohm, Burhap, and Massey [1949] have shown that the ions must enter the sheath with an energy of at least $kT_i = kT_e/2$. Following the treatment of this problem by Grard [1965], a one dimensional plasma of electrons of average energy kT_e and ions of energy kT_i and unperturbed number density n_o is assumed. If the potential of the probe is V_p and the ions are emitted with some velocity at the sheath edge we have, using the equation of continuity and the conservation of energy,

$$n_i = n\sqrt{\frac{kT_i}{kT_i - eV}} \tag{6}$$

where n_i and V are functions of position between the probe and the sheath boundary. Assuming the velocity distribution to be a Maxwellian, that is, no absorption at the probe surface, we have for the electron density

$$n_e = n\,\exp\left(\frac{eV}{kT_e}\right). \tag{7}$$

An estimate on the magnitude of error involved in making this approximation can be made if we assume that electrons with velocities such that $1/2\ mV^2 > e(V-V_p)$ are absorbed by the probe and removed from the distribution function. Then we can write

$$n_e = n\,\exp(eV/kT_e)\int_{-\infty}^{\infty} f(V)\,dV \tag{8}$$

where $$f(V) = \left(\frac{m}{2\pi kT_e}\right)^{1/2} \exp\left(-\frac{mV^2}{2kT_e}\right) \tag{9}$$

thus

$$n_e = n\,\exp(eV/kT_e)\left(\frac{m}{2\pi kT_e}\right)^{1/2}\left[\int_{-\infty}^{0} \exp\,\frac{-mV^2}{2kT_e}\,dV + \int_{0}^{\sqrt{\frac{2e}{m}(V-V_p)}} \exp\left(\frac{-mV^2}{2kT_e}\right) dV\right] \tag{10}$$

or

$$n_e = n/2\ \exp(eV/kT_e)\left[1 + \mathrm{erf}\sqrt{\frac{e(V-V_p)}{kT_e}}\right] \tag{11}$$

we see that at the surface of the probe

$$n_e = n/2\ \exp(eV_p/kT_e) \tag{12}$$

if we let $-eV_p \geq 3kT_e$ we have $n_e \cong 0.025n$, and less than 2.5% of the electrons are collected by the probe. Thus the approximation of a Maxwellian distribution is adequate for negative potentials such that $V_p \leq -3kT_e/e$. Now substituting into Poisson's equation in one dimension,

$$\frac{\partial^2 V}{\partial x^2} = \frac{-e}{\varepsilon_o}(n_i - n_e) \tag{13}$$

we have

$$\frac{\partial^2 V}{\partial x^2} = -\frac{n_e}{\varepsilon_o}\left[\sqrt{\frac{kT_i}{kT_i - eV}} - \exp\left(\frac{eV}{kT_e}\right)\right]. \tag{14}$$

Letting $\frac{\partial^2 V}{\partial x^2} = \frac{\partial}{\partial V}\left(\frac{\partial V}{\partial x}\right)\frac{\partial V}{\partial x}$

and integrating we have,

$$\frac{1}{2}\left(\frac{\partial V}{\partial x}\right)^2 = \frac{n_e}{\varepsilon_o}\left\{\frac{4}{e}kT_i\sqrt{1 - \frac{eV}{kT_i}} - \frac{4kT_i}{e} - \frac{2kT_e}{e}\left[1 - \exp(eV/kT_e)\right]\right\} \tag{15}$$

where the constant of integration has been evaluated by requiring $\partial V/\partial x = 0$. At the sheath boundary $eV/kT_i \ll 1$ and $eV/kT_e \ll 1$. Making the appropriate expansions we obtain

$$\left(\frac{\partial V}{\partial x}\right)^2 = \frac{n_e}{\varepsilon_o}\left[\frac{-e}{2kT_i} + \frac{e}{kT_e}\right]V^2. \tag{16}$$

In order for $(\partial V/\partial x)^2$ to be positive, that is $\partial V/\partial x$ a real number, $e/kT_e > e/2kT_i$ must be satisfied or $kT_i > kT_e/2$. Thus the ions must cross the sheath boundary at velocities greater than zero and in most cases greater than their thermal velocities, since in most laboratory plasmas $T_i \ll T_e$.

We note making use of the Child-Langmuir law for the probe current that the thickness of the sheath may be expressed (see Equation 5) as

$$d = \frac{2}{3}\left(\frac{\varepsilon_o A}{I_p}\right)^{1/2}\left(\frac{2e}{M_i}\right)^{1/4}\left(-V_p\right)^{3/4} \tag{17}$$

Neglecting variations in ion current due to changes in probe potential, we note that the thickness of the sheath increases with decreasing probe potential. If one considers a model of the sheath as having a distinct boundary, one can imagine the sheath boundary and the probe to be a plane parallel capacitor. As the probe voltage decreases, the sheath grows thicker, the capacity decreases, and the sheath impedance at a given frequency should increase.

A more accurate description of the sheath impedance was given by Crawford and Grard (1966). If we look at the integrated form of Poisson's equation (see Equation 15),

$$\frac{\partial V}{\partial x} = \left(\frac{n_e}{\varepsilon_o}\right)^{1/2}\left\{\frac{4kT_i}{e}\sqrt{1-\frac{eV}{kT_i}} - \frac{4kT_i}{e} - \frac{2kT_e}{e}\left[1-\exp\left(\frac{eV}{kT_e}\right)\right]\right\}^{1/2} . \tag{18}$$

Evaluating this at the probe surface and setting $\exp(V_p/V_e) \ll 1$ gives

$$\left(\frac{\partial V}{\partial x}\right)_{x=0} = (kT_e)^{1/2}\frac{2}{e\lambda_D}\left[kT_i\sqrt{1-\frac{eV_p}{kT_i}} - \left(kT_i + \frac{kT_e}{2}\right)\right]^{1/2} . \tag{19}$$

The capacity of the sheath is defined as

$$C_s = \frac{\partial Q_p}{\partial V_p} \tag{20}$$

The charge Q_p on the probe is given by Gauss' theorem as $Q_p = A\varepsilon_o E_p$ where E_p is the electric field at the surface of the probe. Since

$$E_p = -\left(\frac{\partial V}{\partial x}\right)_{x=0}$$

in one dimension, we obtain the relation for C_s,

$$C_s = -A\varepsilon_o \frac{\partial}{\partial V_p}\left(\frac{\partial V}{\partial x}\right)_{x=0} \tag{21}$$

Carrying out the differentiation we obtain

$$C_s = \frac{\varepsilon_o A}{\lambda_D}\,\frac{1}{2}\left[\frac{T_i}{T_e}\left(1-\frac{eV_p}{kT_i}\right)^{3/2} - \left(\frac{T_i}{T_e}+\frac{1}{2}\right)\left(1-\frac{eV_p}{kT_i}\right)^{1/2}\right]^{-1/2} \tag{22}$$

Assuming the ions enter the sheath with energy $kT_e/2$ we can write

$$C_s = \frac{\varepsilon_o A}{\lambda_D}\;\frac{1}{2}\left[\frac{1}{2}\left(1-\frac{2eV_p}{kT_e}\right)^{3/2} - \left(1-\frac{2eV_p}{kT_e}\right)^{1/2}\right]^{-1/2} \tag{23}$$

giving C_s as a function of electron temperature, number density, probe potential, and probe dimensions. A plot of C_s as a function of probe potential is shown in Figure 15.

The AC equivalent circuit is shown in Figure 2. Z_{s1} is the sheath impedance dure to the sheath on the transmitting probe. Z_{s3} represents the sheath impedance at the receiving probe and Z_{s2} represents the sheath impedance due to the sheaths at every other point in the chamber. Z_{p1} and Z_{p2} represent impedances in the plasma coupling the sheath impedances together.

When the probe separation is fixed and the bias voltage is varied Z_{p2} and Z_{s2} remain essentially constant This is due to the short Debye length in the plasma. Effects due to the sheaths can be estimated from Equation 17. If the sheath dimensions are small compared to the probe separation the response should be essentially the response due to the change in the sheath impedance as a function of bias voltage. This assumes that the input impedance of the oscilloscope in series with the impedance of the sheath on the receiving probe is much larger than the plasma impedance. This is necessary in order to avoid drawing currents through the receiving probe large enough to disturb the plasma at the point being measured. The input impedance of the oscilloscope is 10^8 ohms. Since the oscilloscope draws little current, it will be assumed that the probe is at floating potential, that is, that potential at which no current is drawn by the probe. It can be shown as by Grard (1965) that the floating potential is given by

$$V_f = \frac{-kT_e}{2e} \ln \frac{M_i}{2\pi m_e}.$$

For argon $V_f = -4.68V_e$. Using this potential in the expression for the sheath capacitance we obtain a value of 3.1×10^{-13} farads which gives an impedance of 3.4×10^8 ohms.

The impedance through the plasma is unknown. The dielectric constant for the plasma is $\kappa = 1 - (\omega_p/\omega)^2$. At frequencies of 1.5×10^4 Hz and plasma frequency of 1.8×10^8 Hz, $\kappa = 1.44 \times 10^8$. This suggests that the capacitances between the probes is increased by a factor of 10^8 when the plasma is present. Since without the plasma, the capacitances will be on the order of a few pico-farads or less, the impedances will be on the order of 100 ohms with the plasma in the chamber. The frequency dependence of the dielectric constant suggests that the frequency response of the total system should decrease as the frequency of the transmitted signal increases which is observed to happen.

We can write the voltage observed on the oscilloscope as

$$V' = V_{in}\left(\frac{10^8\Omega}{Z_{s3}+10^8\Omega}\right)\left(\frac{Z_{p2}+Z_{s2}}{Z_{s1}+Z_{p1}+Z_{s2}+Z_{p2}}\right) \tag{24}$$

Ignoring the plasma impedances with respect to the sheath impedances, assuming $Z_{s3} = 3.4 \times 10^8$ ohms, and assuming $Z_{s2} = Z$, a constant value, and all impedances to be capacitive, we have

$$V' = .282V_{in}\left(\frac{Z}{\frac{1}{\omega C_s}+Z}\right)$$

rewriting this we have

$$V' = .282V_{in}\left(\frac{Z}{1+\frac{Z}{Z_{s1}}}\right)\omega C_s \tag{25}$$

From equation 23 for C_s we note that C_s is directly proportional to the area of the probe, and thus that the impedance due to the sheath is inversely proportional to the probe area. Since the area of the "probe" with impedance Z in the above formula is the area of everything else in the chamber other than the receiving and transmitting probes, $Z << Z_{s1}$ and we can write $V' = KC_s$ where C_s is the capacity of the transmitting probe's sheath and K is a constant, $K = .282\omega ZV_{in}$.

The response of the receiving probe for a probe separation of 4.0 cm at the fundamental frequency is plotted on Figure 15, along with the theoretical response of the sheath capacitance. It can be seen that the striking deviation is the minimum in the received signal, which does not occur in the theoretical curve.

Computing the sheath dimensions using Equation 17 for the data used to obtain Figure 14 gives sheath dimensions at the transmitting probe of 2.5 cm. The sheath dimensions at the transmitting probe tend to remain fairly constant as a function of bias voltage, changing by only three millimeters over the entire range of bias voltage.

It is difficult to say how meaningful this number is, since it is apparent that the Child-Langmuir law will not be valid for predicting sheath dimensions without knowledge of the effect which the receiving probe will have on the sheath dimensions. A detailed analysis of the potential as a function of distance between the two objects is needed to predict what the sheath dimensions are.

The observed data, in fact, would seem to indicate that the sheath dimensions predicted by the Child-Langmuir law are incorrect. Both changing the bias voltage on the transmitting probe and changing the separation between the two probes produces a similar effect. This would seem to indicate that the nonlinearities observed are caused by some kind of interaction between the sheath surrounding the transmitting probe and the sheath surrounding the receiving probe.

If the Child-Langmuir law gives the correct sheath dimensions, the potential minimum in the magnitude of the a. c. component of the received signal would occur at different positions in the sheath for different values of bias voltage. The sheath dimensions themselves would remain essentially constant with changing bias voltage. This seems unreasonable and would suggest that the sheath dimensions predicted by Equation 17 are invalid.

VI. POSSIBILITIES FOR PRODUCTION OF MINIMUM AMPLITUDE IN FUNDAMENTAL FREQUENCY OF RECEIVED WAVE FORM

It is difficult to say what phenomenon is responsible for the production of the observed minimum in the signal observed with the receiving probe. One possibility is that the sheath, in its motion following the applied AC signal, produces a time dependent potential which influences the motion of thermal particles approaching the sheath boundaries. A particle coming into the area surrounding the probe with insufficient energy to overcome the potential barrier to the probe must slow down, stop, and be reflected from the probe. If while in the potential field of the probe, the potential changes by the mechanism of the AC potential variation applied to the probe, then the particle may be reflected with more, or less, energy than it entered with. Large numbers of these particles could produce number density fluctuations in the plasma propagating at slower velocities than the electro-magnetic coupling between the two probes. A similar phenomena has been investigated by Lonngren and Montgomery [1967] in connection with the propagation of ion sound waves. These "pseudo waves" are bursts of ions which resemble ion sound waves, but which have a different velocity. They were noted as ion bursts emerging from a grid which had a step voltage applied to it to produce a pulse propagating in the ion sound wave mode. The ions moving in the vicinity of the grid suddenly gained energy as the potential was charged and showed up as "bumps" in the potential at the receiving probe. In this case, the fundamental frequency could be reaching a minimum because of the arrival of bursts of particles out of phase with the original signal. The particles in this case would have to be electrons because the probe is collecting ion current. It seems very unlikely that such a process would occur. The thermal velocities of electrons at 0.35 electron volts are 4×10^5 m/sec. Assuming no time variation of potential from the sheath to the plasma, and assuming a potential of the form

$$\phi(x) = V_p \exp(-x/\lambda_D) \tag{26}$$

for the sheath potential, an expression was obtained for the time necessary for an electron to travel a length of 10 Debye lengths, and be reflected back to its original position. The expression obtained was

$$\tau = \frac{2\lambda_D}{v_{th}} \ln\left\{\frac{1+\left[1-\frac{eV_p}{kT_e}\exp(-x/\lambda_D)\right]^{1/2}}{1-\left[1-\frac{eV_p}{kT_e}\exp(-x/\lambda_D)\right]^{1/2}}\right\} \tag{27}$$

where $x = 10\lambda_D$

$V_p = 55$ volts

$v_{th} = 4 \times 10^5$ meters/second

$\lambda_D = 1.4 \times 10^{-3}$ meters

The time, τ, calculated for the above parameters was $\tau = 4.4 \times 10^{-8}$ seconds. This time is only a fraction of the period of the AC variation in potential which is 6.7×10^{-5} seconds. It would seem far more likely that the ions would be able to produce such an effect. Assuming the ions to be at room temperature, and using their thermal velocity at that temperature, $\tau = 1.4 \times 10^{-5}$ seconds. This is close to the period of the AC input voltage, but the fact remains that the probe is at a negative voltage and drawing ion current. This would seem to indicate that the ions would be attracted from the plasma to the negatively biased plate. One possibility may be that the ions and electrons are not emitted into the sheath with zero velocity as is assumed. The effect of a finite velocity may be to produce changes in the potential near the edge of the sheath which overshoot zero before emerging into the plasma. In such a case the ions could be reflected off such an overshoot and possibly cause such an effect instead of being attracted to the transmitting plate.

An alternative explanation is that this phenomenon is similar to an experiment done by von Gierke, et al [1961].

This experiment was done in rf excited plasmas. It was found that if a microwave was propagated through the plasma at a frequency f, the fundamental was modulated at the rf frequency. Side-bands appeared at $f \pm f_1$, $f \pm 2f_1$, $f \pm 3f_1$, etc., where f_1 was the frequency of the rf excitation. The explanation taken from Heald [1965] was that the index of refraction in the plasma varied as a function of time with the number density. A perturbation solution of the wave equation was done which showed solutions giving the observed modulation. The possibility existed in the solutions for the amplitude of the fundamental frequency to go to zero, as it was for all the sidebands also. Although this problem is considerably different from the experiment described here, it is a possibility that a similar situation could produce the effects observed. The AC signal could change the number density in the sheath region, which would change the refractive index, and, perhaps, modulate the wave at its own frequency. This would produce the fundamental, the second harmonic, third harmonic, and so forth. The problem would be difficult to analyze since the refractive index would vary in a complicated manner as a function of both time and position.

A third possibility is that there may be reflections off of the inside of the chamber setting up standing waves. It is not clear exactly how such a process would happen, but it is a possibility.

Finally, there may be an interaction of the sheath edge of the transmitting probe with the sheath edge of the receiving probe. This would seem to be the most reasonable type of mechanism for the following reasons. No anomalies are observed to occur until the transmitting probe and the receiving probe are within an order of magnitude estimate of the sheath dimensions. The similarity in the received signal as a function of changes in the bias voltage on the transmitting probe and as a function of changes in the separation of the two probes would seem to indicate that the nonlinearities observed are caused by some kind of interaction between the sheaths of the receiving and the transmitting probes. One would expect sheath dimensions to change with bias voltage in the manner observed. The minimum in the received signal occurs at larger probe separations for more negative values of bias voltage. This would seem to indicate that sheath dimensions increase with more negative values in bias voltage, which is what one would expect to happen. One could imagine a resonance occuring between the edges of the two sheaths which produce the nonlinearities seen in the received signal. It is not really proper, however, to think of the sheath as having definite boundaries. It is actually a continuous variation in number density, potential, etc., which has effects measurable only over several Debye lengths. An accurate description of the potential between the two probes, and the reaction of large numbers of particles in that potential will prove to be difficult.

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Lonngren, K., D. Montgomery, I. Alexeff, and W. D. Jones, Dispersion of ion-acoustic waves, Phys. Letters, 25A(8), 629-630 (1967).

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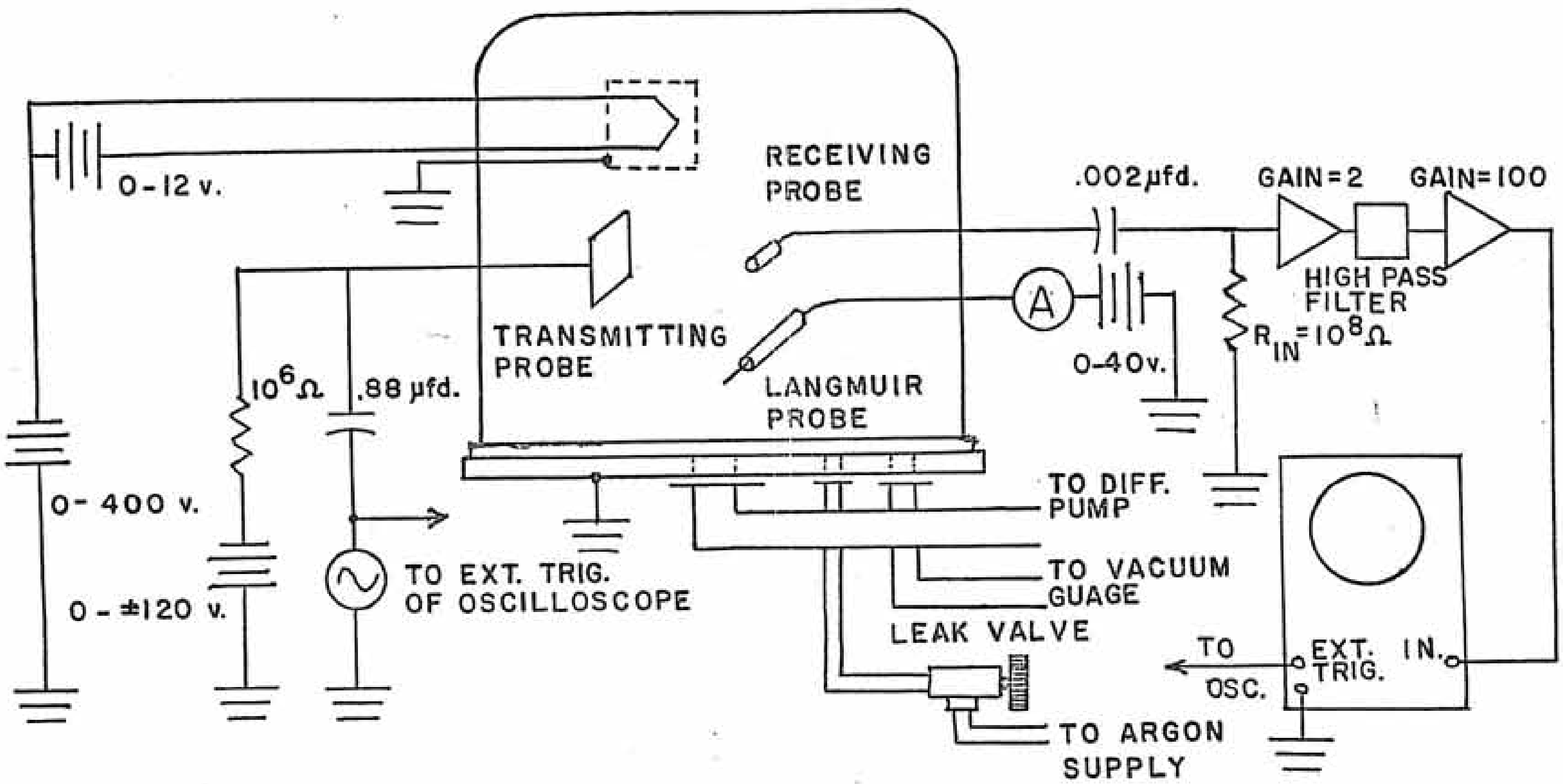
Von Gierke, G., L. Lisitano, G. Muller, H. Schluter, M. Tutter, and H. Wulff, (1961), Vergleidi spektroschopische Unterouchen mit mikrowellen-und sonder, Messungen. Proceedings of the 5th International Conference on Ionization Phenomena in Gases (Munich), 380, Amsterdam, North-Holland Publishing Co., (1962).

Figure 1. Schematic diagram of apparatus.

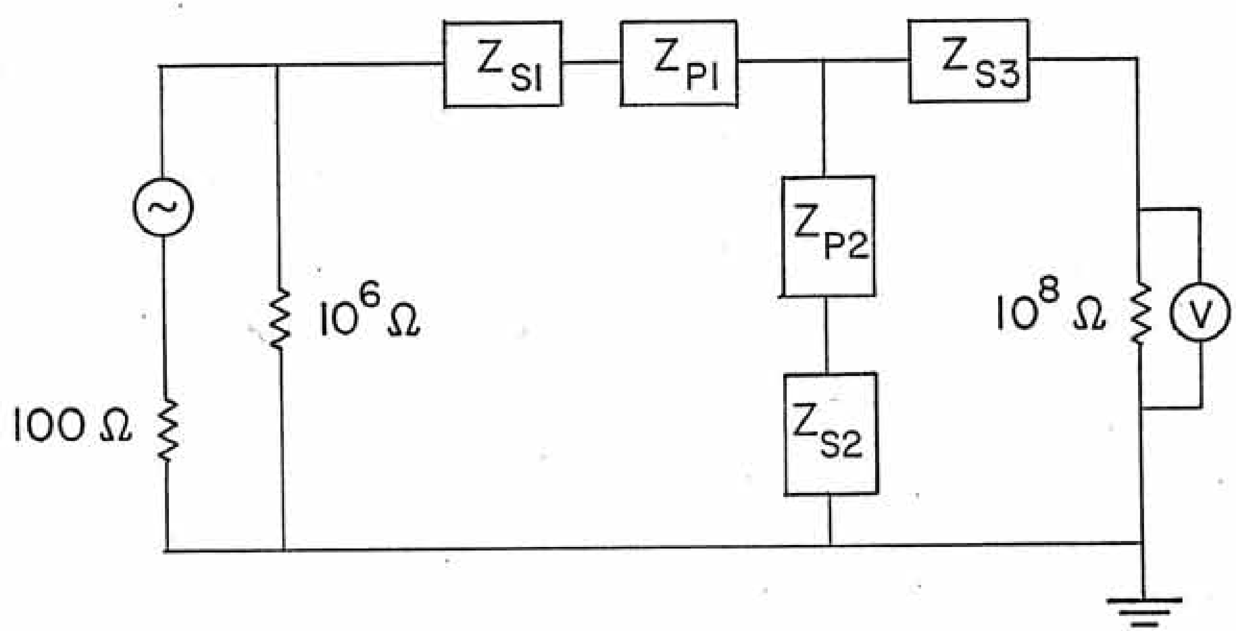


0-12 v.
RECEIVING PROBE
.002 μfd.
GAIN=2
GAIN=100
HIGH PASS FILTER
R IN = 10^8 Ω
TRANSMITTING PROBE
LANGMUIR PROBE
0-40v.
10^6 Ω
.88 μfd.
0- 400 v.
0 - ±120 v.
TO EXT. TRIG. OF OSCILLOSCOPE
TO DIFF. PUMP
TO VACUUM GUAGE
LEAK VALVE
TO OSC.
EXT. TRIG.
IN.
TO ARGON SUPPLY

FIGURE I

Figure 2. AC equivalent circuit for measuring probes.

A-G69-552



EQUIVALENT AC CIRCUIT

FIGURE 2

Figure 3. Photograph of plasma chamber in open position.

Figure 4. Photograph of plasma chamber in closed position.

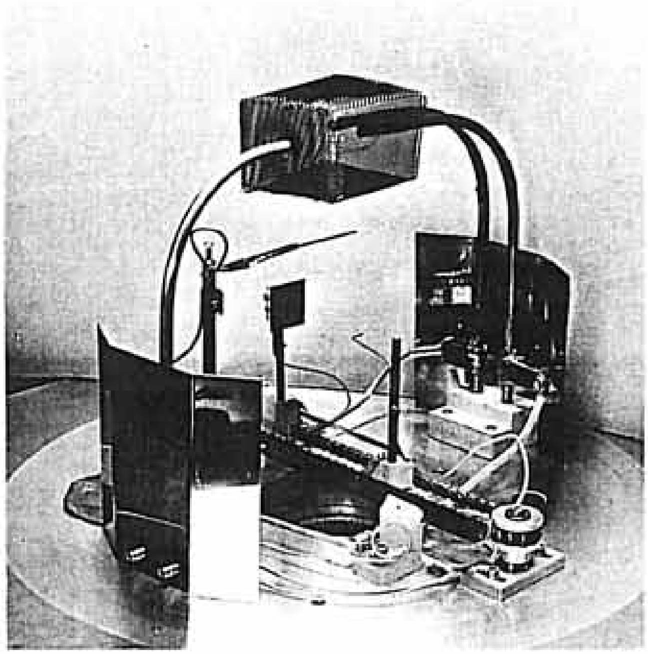


FIGURE 3

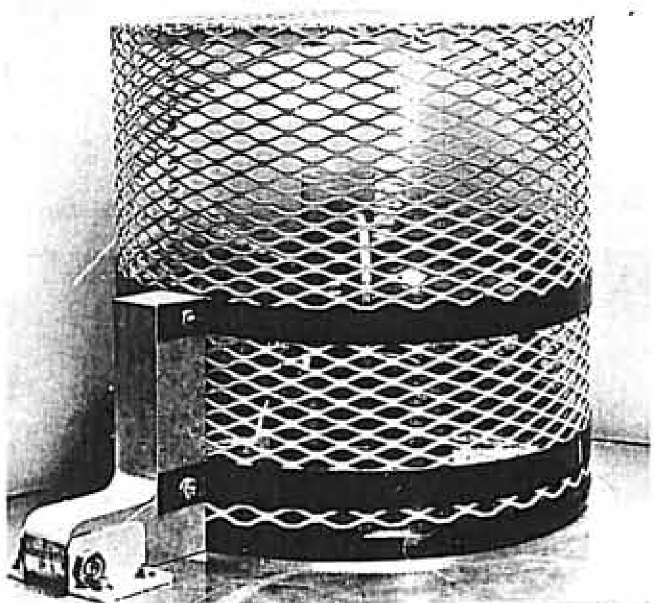


FIGURE 4

Figure 5. Typical Langmuir probe characteristic.

G69-576 A

LANGMUIR PROBE CHARACTERISTIC

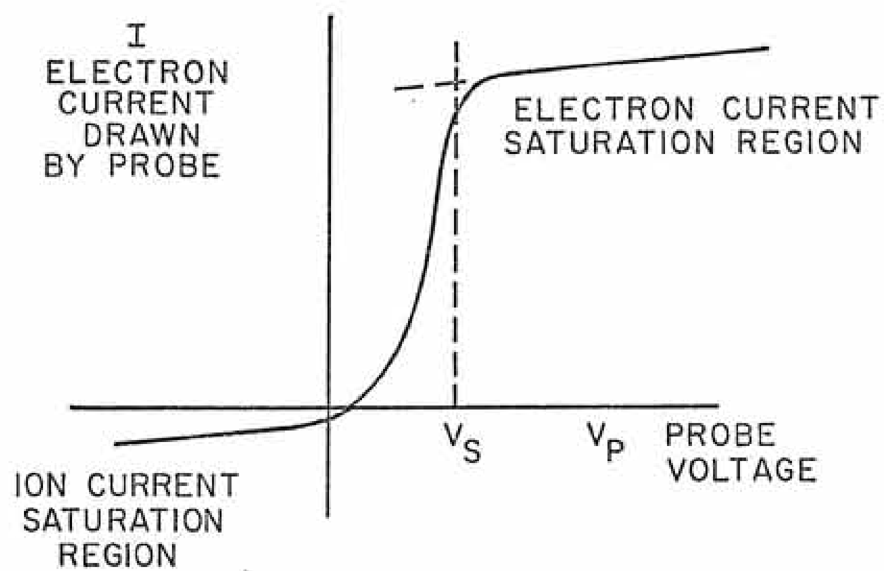


FIGURE 5

Figure 6. Photographs of change in received signal due to variation of probe separation at constant bias voltage.

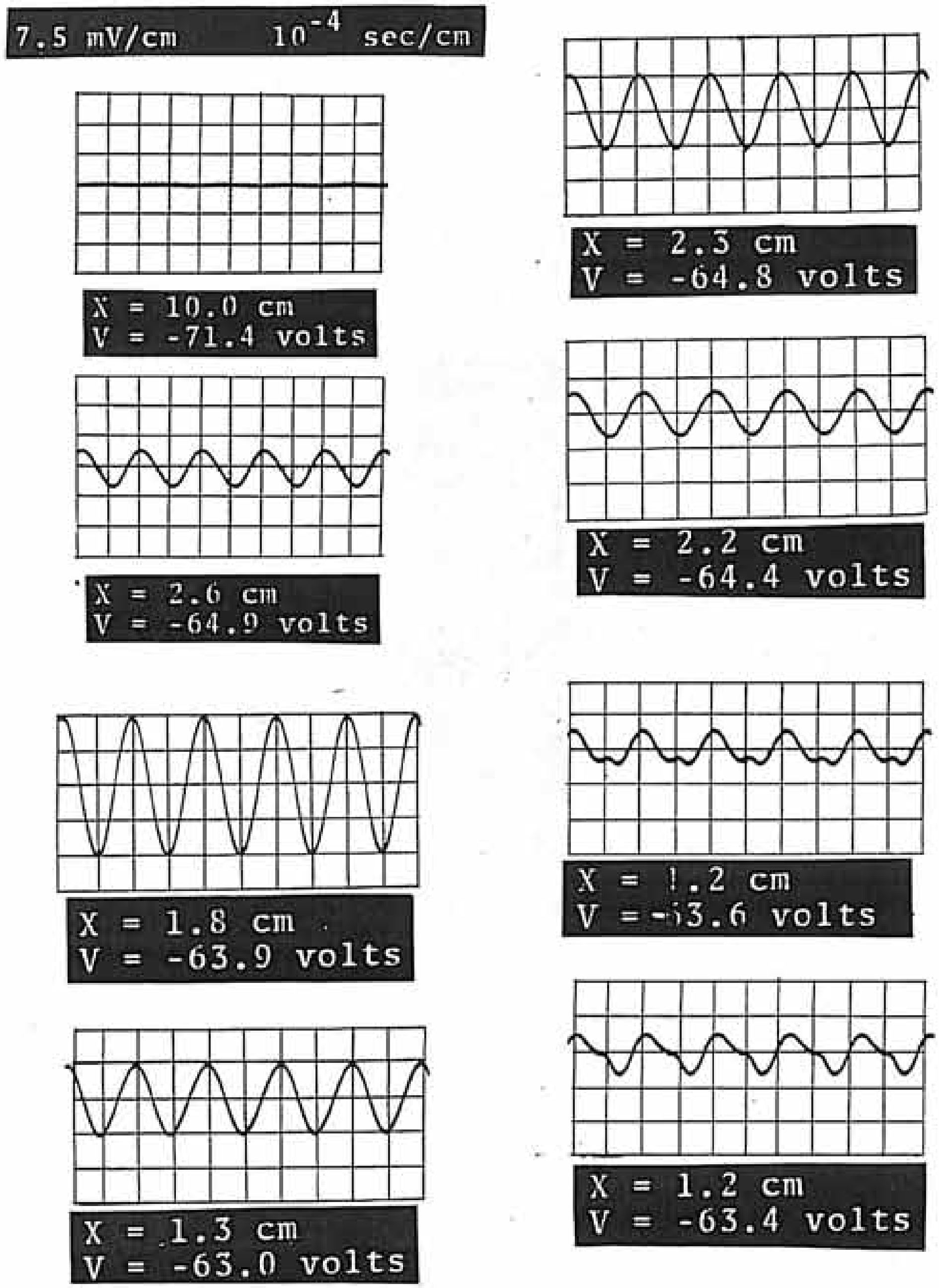

7.5 mV/cm 10^{-4} sec/cm
X = 10.0 cm
V = -71.4 volts
X = 2.3 cm
V = -64.8 volts
X = 2.6 cm
V = -64.9 volts
X = 2.2 cm
V = -64.4 volts
X = 1.8 cm
V = -63.9 volts
X = 1.2 cm
V = -63.6 volts
X = 1.3 cm
V = -63.0 volts
X = 1.2 cm
V = -63.4 volts

FIGURE 6

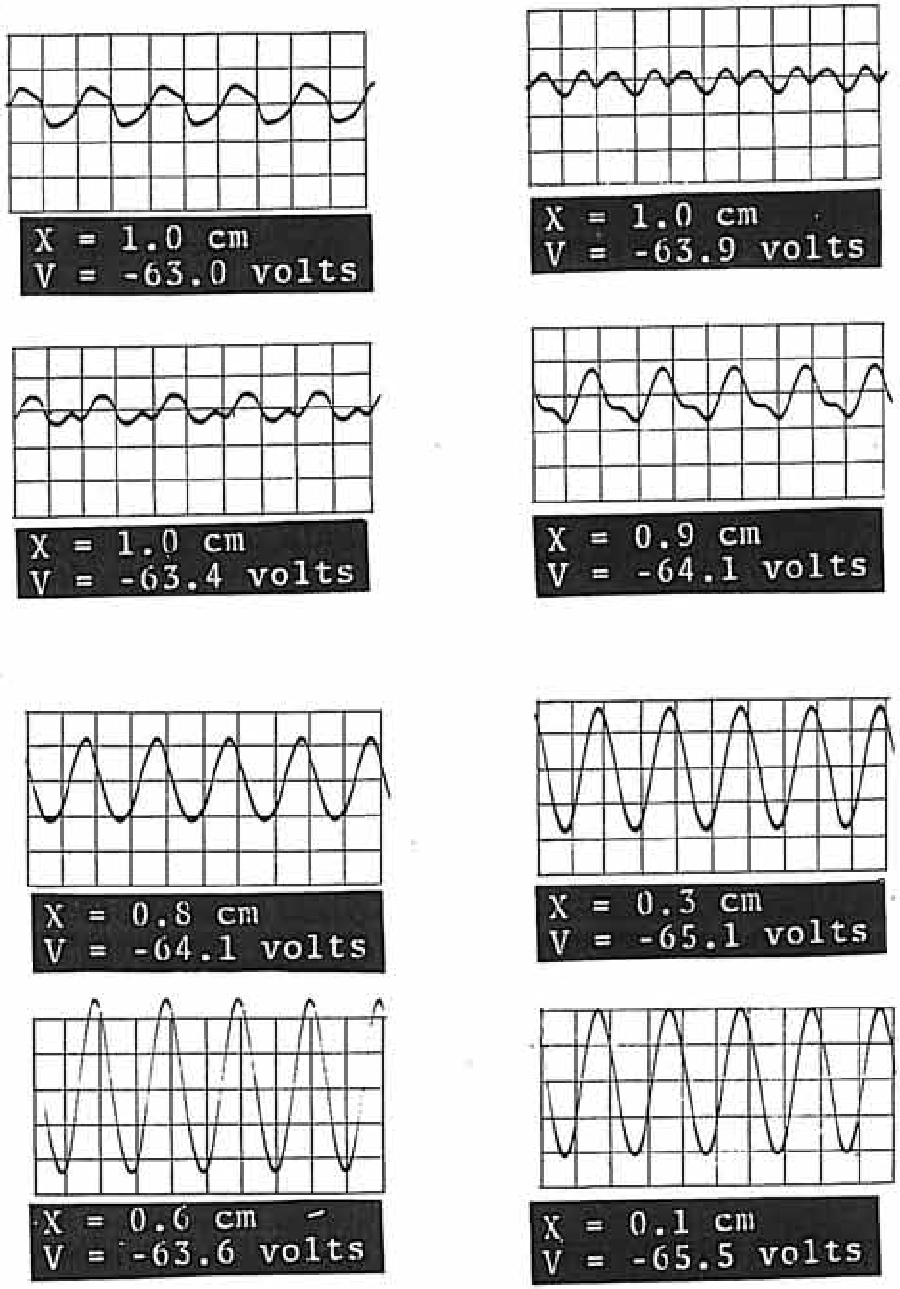
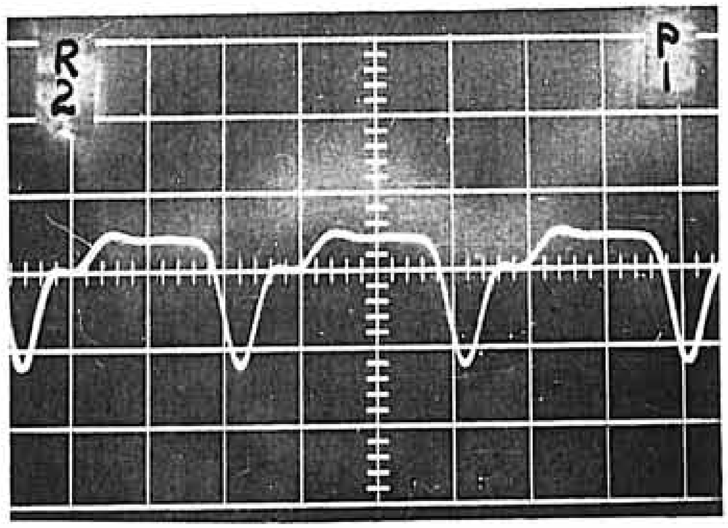


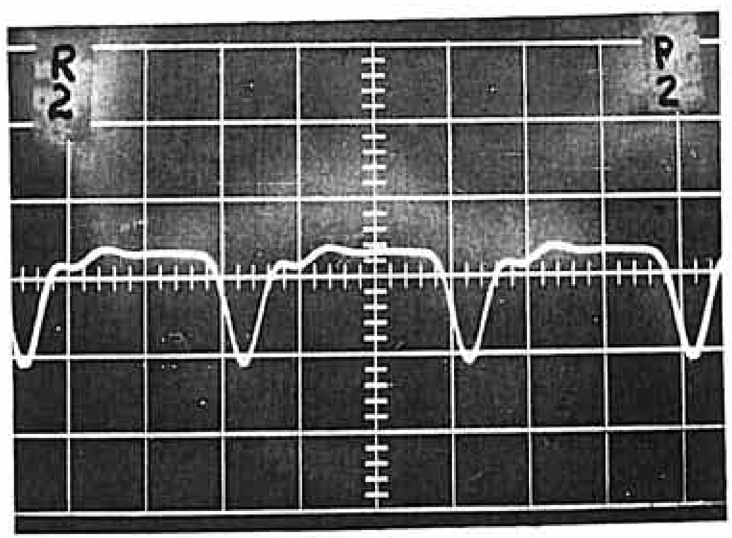
FIGURE 6 (Con't.)

Figure 7. Photographs of change in received signal due to variations of bias voltage at constant probe separation.

PLATE SEPARATION = 4.0 CM
INPUT SIGNAL FREQUENCY = 15 kHz

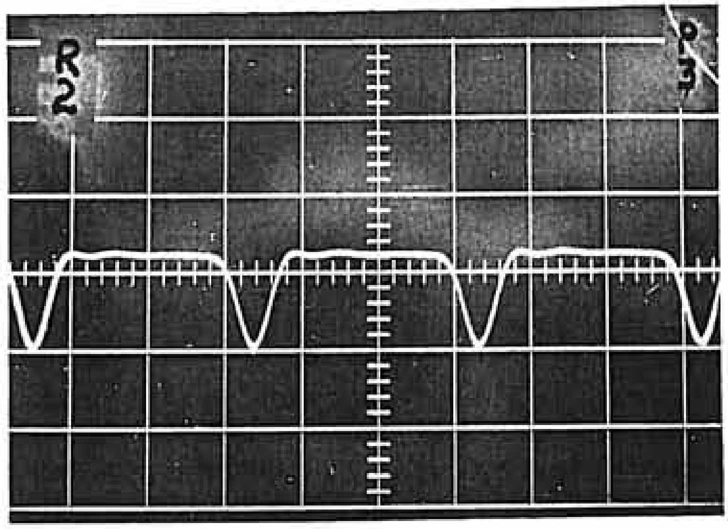


$I_p = -122.31$ μamps
37.5 mv/cm

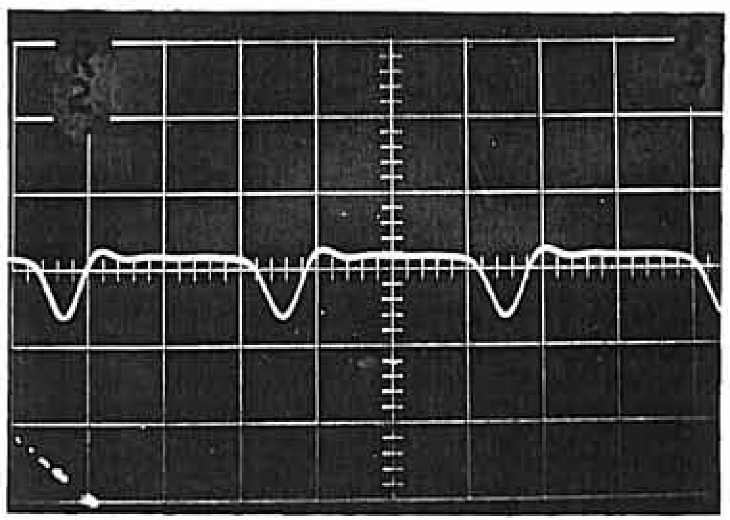


$I_p = -98.28$ μamps
37.5 mv/cm

FIGURE 7

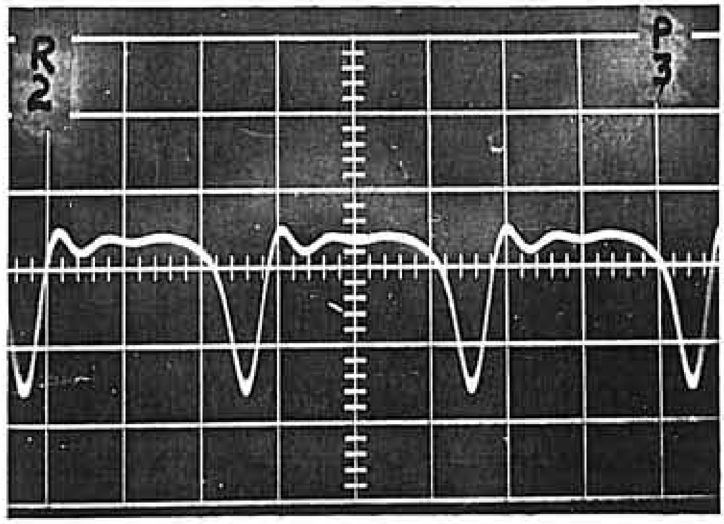


I_p = -68.69 μamps
37.5 mv/cm

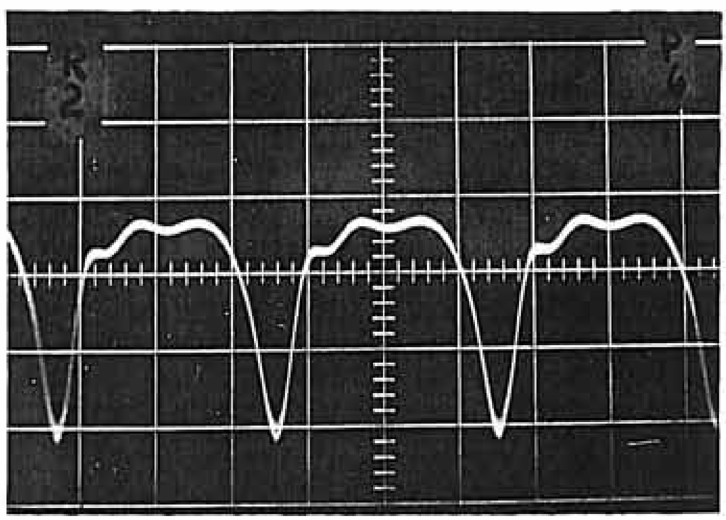


I_p = -43.53 μamps
37.5 mv/cm

FIGURE 7 (Con't.)

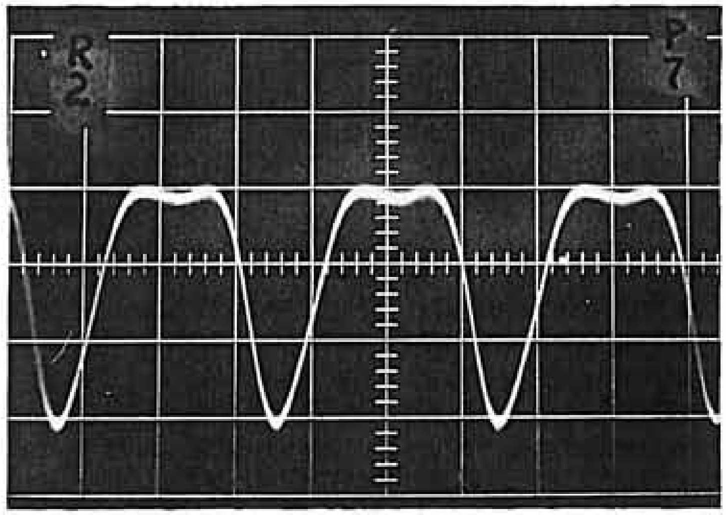


I_p = -16.39 μamps
3.75 mv/cm

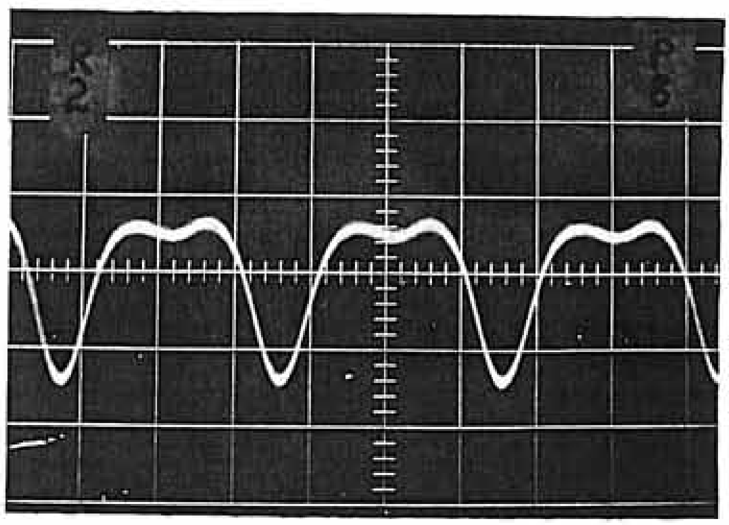


I_p = -11.83 μamps
1.50 mv/cm

FIGURE 7 (Con't.)

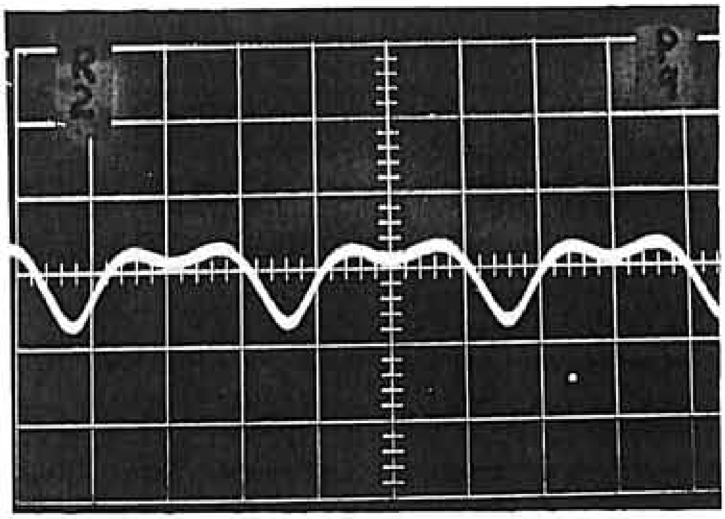


$I_p = -7.71$ μamps
0.75 mv/cm

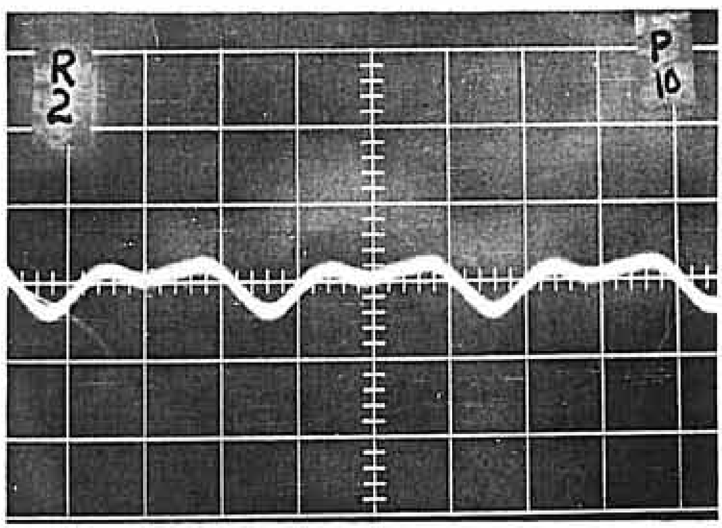


$I_p = 2.23$ μamps
0.75 mv/cm

FIGURE 7 (Con't.)

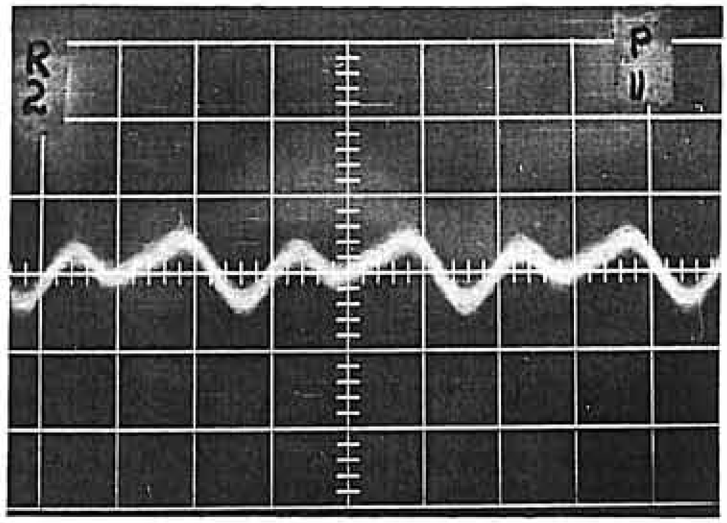


I_p = 6.95 μamps
0.75 mv/cm

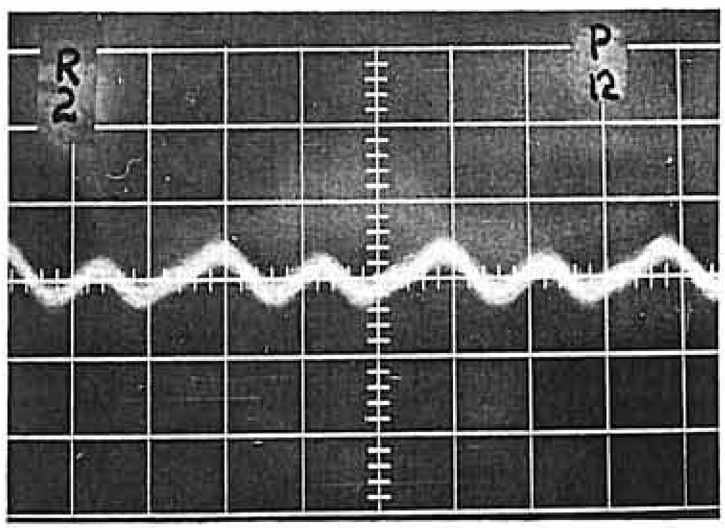


I_p = 8.54 μamps
0.75 mv/cm

FIGURE 7 (Con't.)

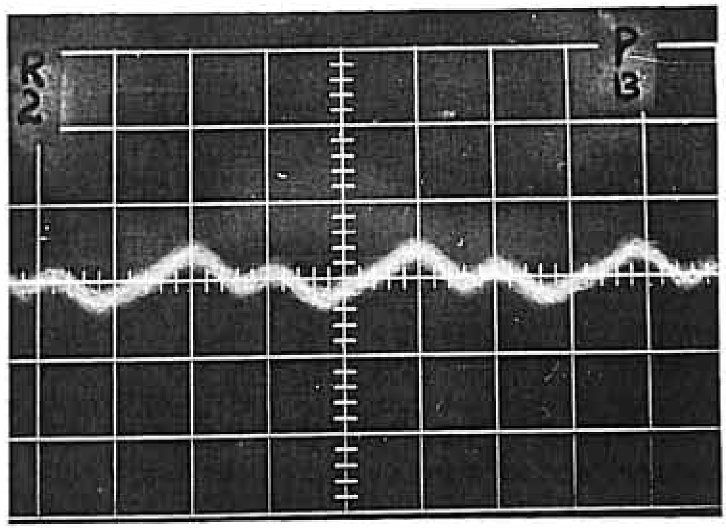


I_p = 10.24 uamps
0.375 mv/cm

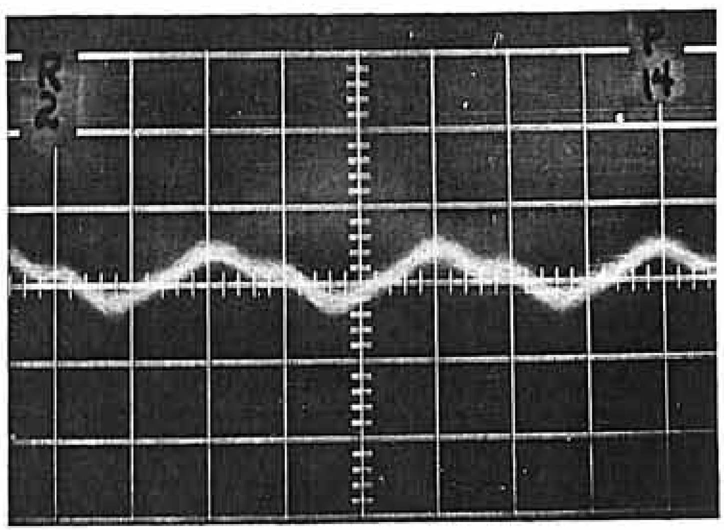


I_p = 11.72 uamps
0.375 mv/cm

FIGURE 7 (Con't.)

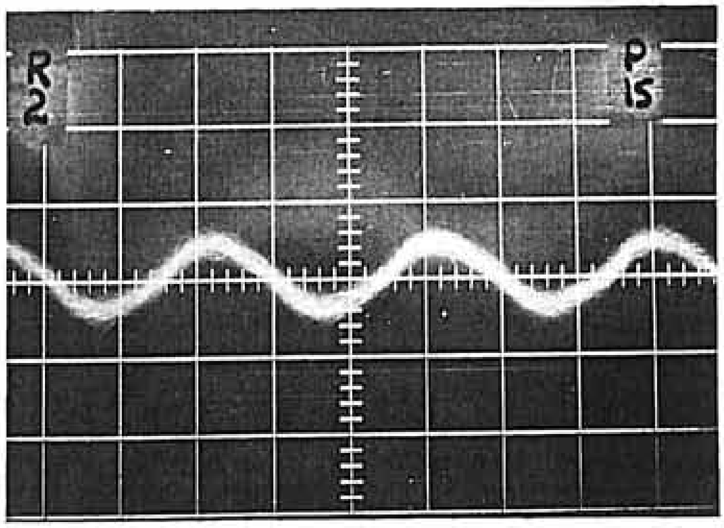


I_p = 13.20 µamps
0.375 mv/cm

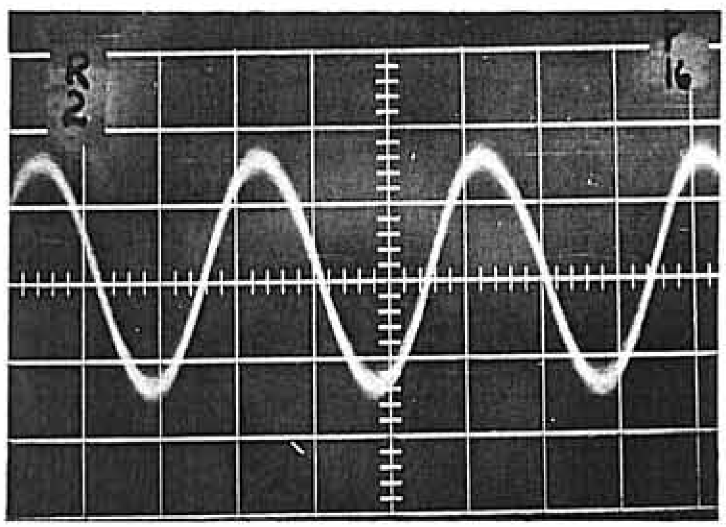


I_p = 14.68 µamps
0.375 mv/cm

FIGURE 7 (Con't.)



I_p = 17.53 μamps
0.375 mv/cm



I_p = 22.56 μamps
0.375 mv/cm

FIGURE 7 (Con't.)

Figure 8. Frequency components of received signal at 1.0 centimeters of probe separation.

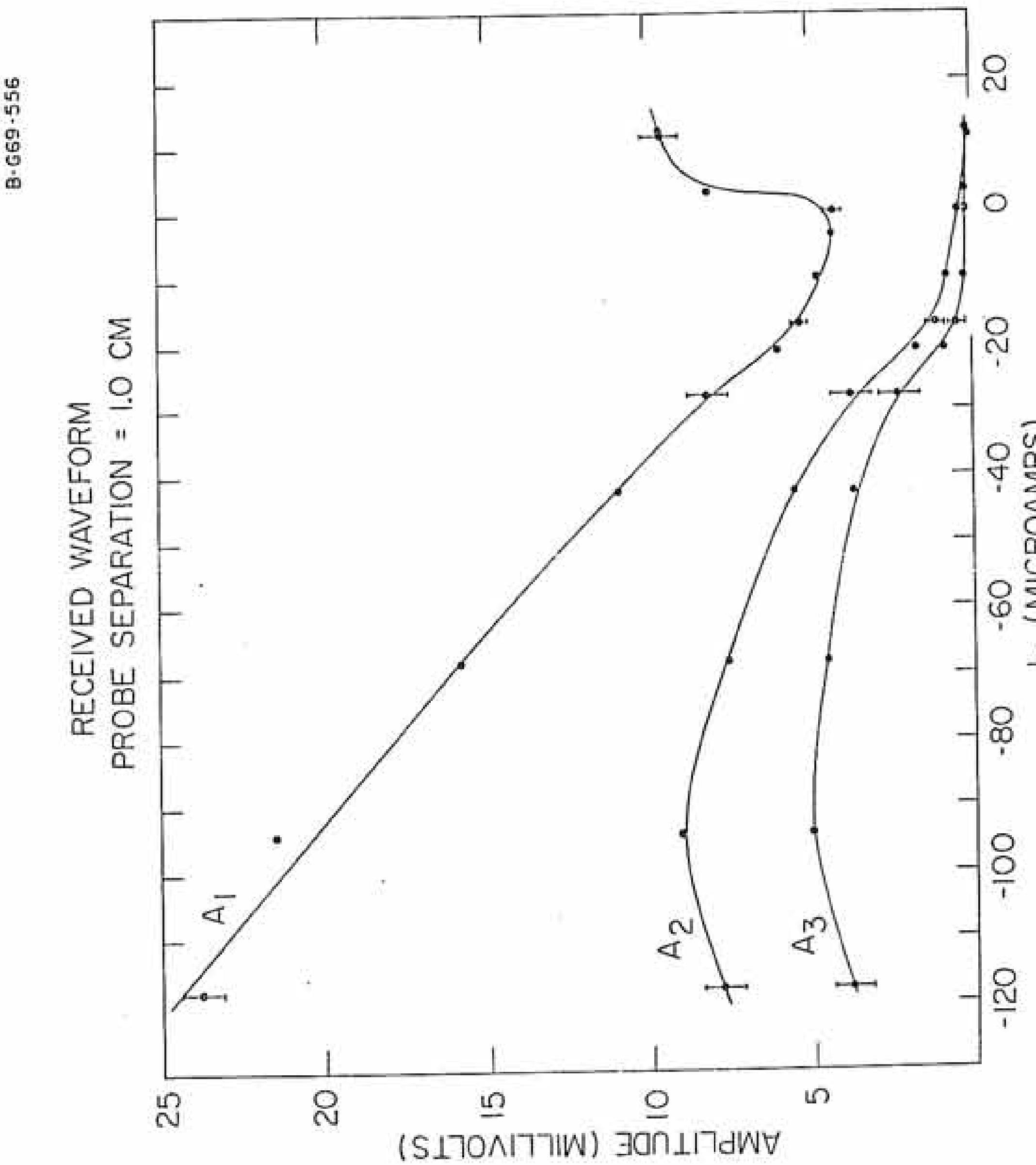

B-G69-556
RECEIVED WAVEFORM
PROBE SEPARATION = 1.0 CM
AMPLITUDE (MILLIVOLTS)
25
20
15
10
5
A1
A2
A3
-120
-100
-80
-60
-40
-20
0
20
Ip (MICROAMPS)

FIGURE 8

Figure 9. Frequency components of received signal at 2.0 centimeters of probe separation.

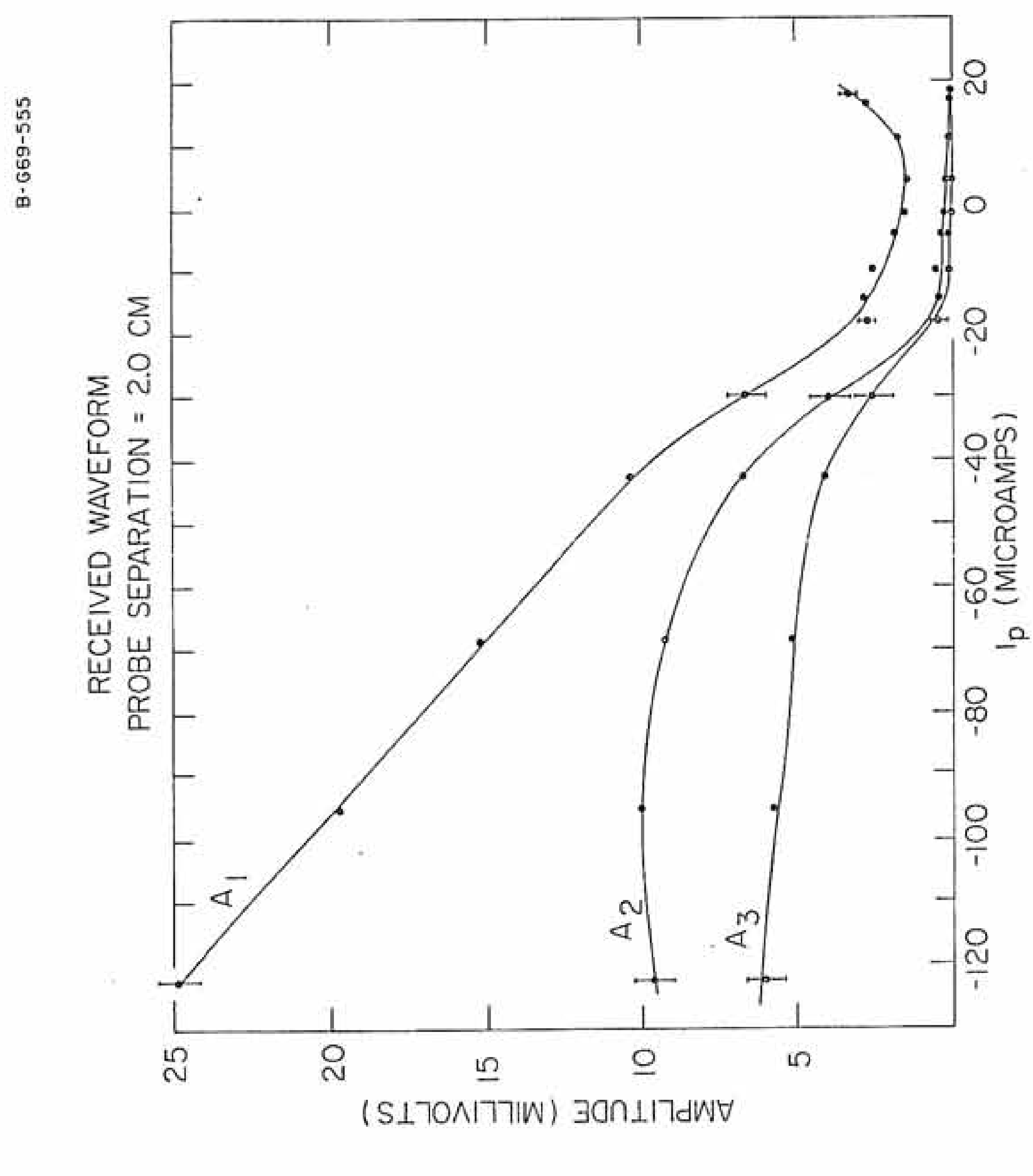

B-G69-555
RECEIVED WAVEFORM
PROBE SEPARATION = 2.0 CM
A1
A2
A3
AMPLITUDE (MILLIVOLTS)
25
20
15
10
5
-120
-100
-80
-60
-40
-20
0
20
Ip (MICROAMPS)

FIGURE 9

Figure 10. Frequency components of received signal at 3.0 centimeters of probe separation.

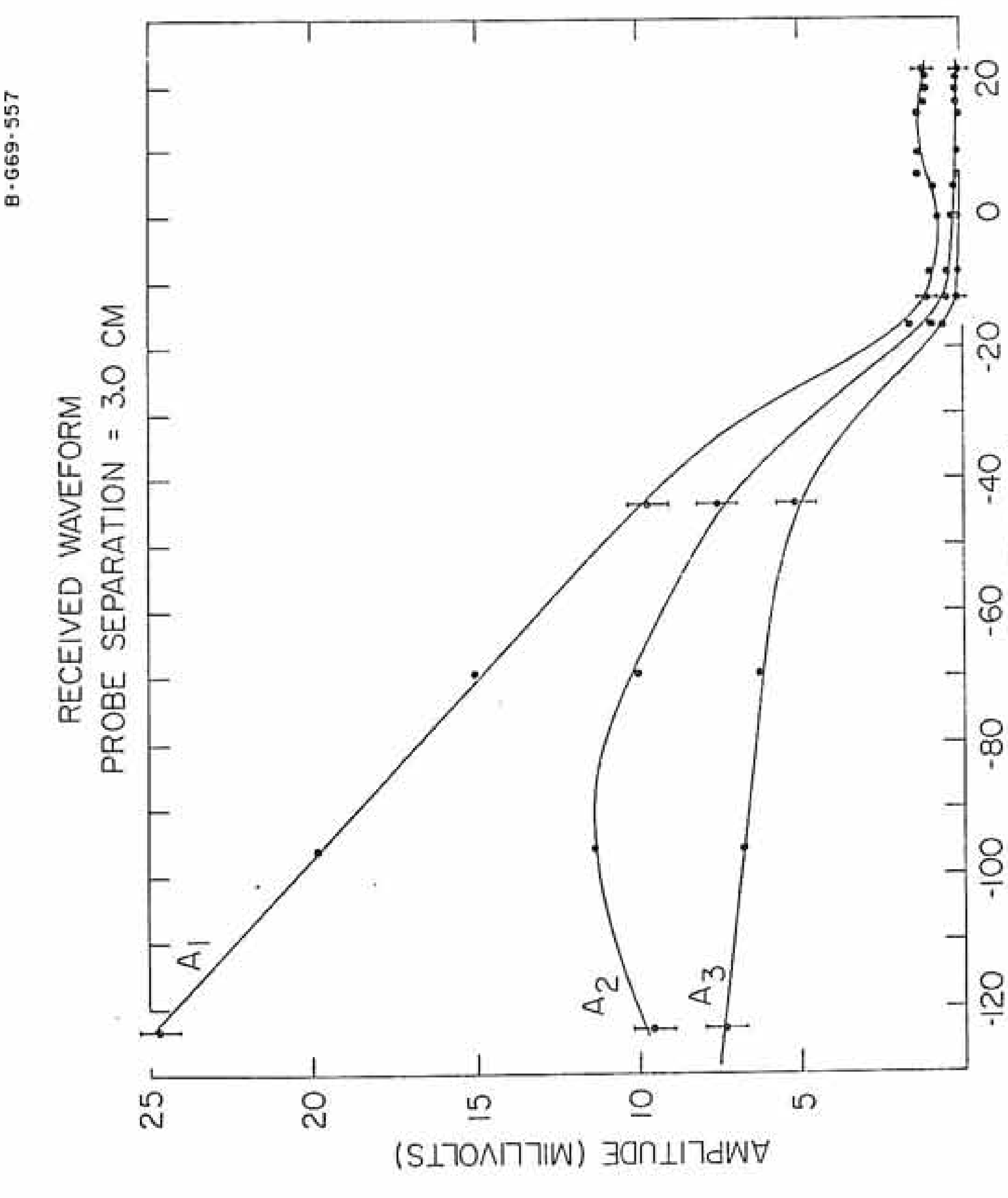

B-G69-557
RECEIVED WAVEFORM
PROBE SEPARATION = 3.0 CM
AMPLITUDE (MILLIVOLTS)
I_p (MICROAMPS)
A_1
A_2
A_3
25
20
15
10
5
-120
-100
-80
-60
-40
-20
0
20

FIGURE 10

Figure 11. Frequency components of received signal at 4.0 centimeters of probe separation.

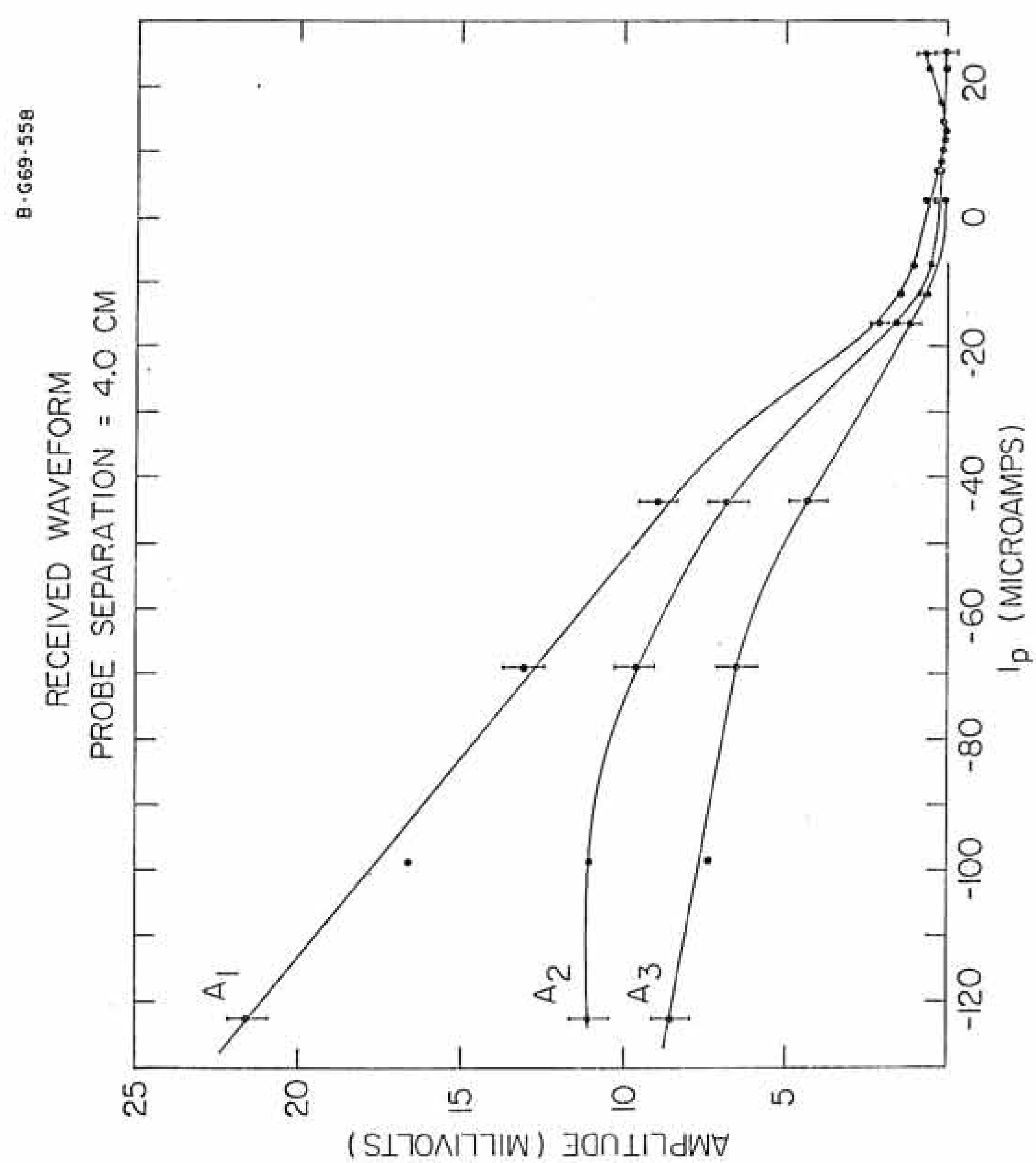

B-G69-558
RECEIVED WAVEFORM
PROBE SEPARATION = 4.0 CM
AMPLITUDE (MILLIVOLTS)
I_p (MICROAMPS)
A_1
A_2
A_3
25
20
15
10
5
-120
-100
-80
-60
-40
-20
0
20

FIGURE 11

Figure 12. Frequency components of received signal at 5.0 centimeters of probe separation.

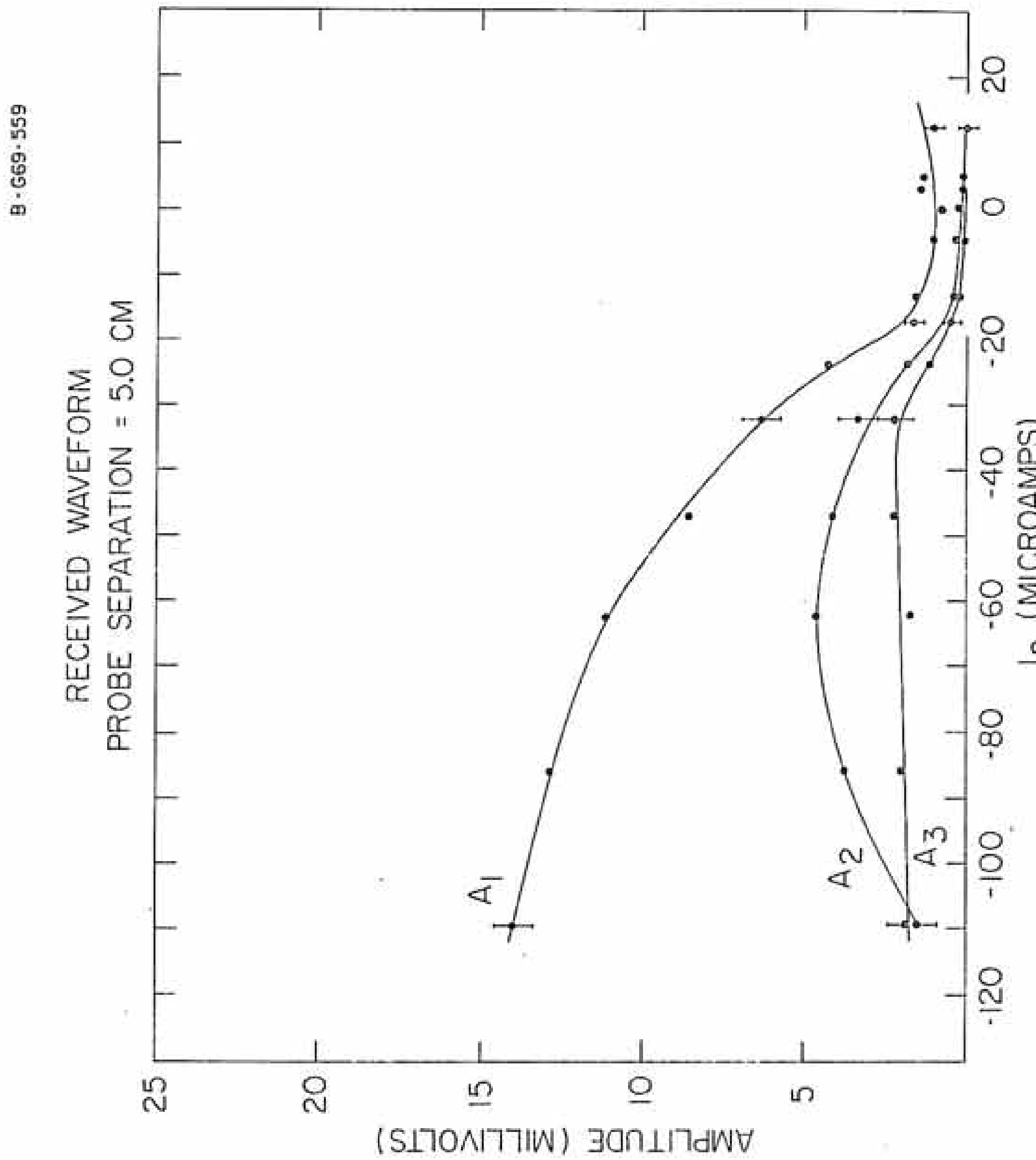

B-G69-559
RECEIVED WAVEFORM
PROBE SEPARATION = 5.0 CM
AMPLITUDE (MILLIVOLTS)
I_p (MICROAMPS)
A_1
A_2
A_3
25
20
15
10
5
-120
-100
-80
-60
-40
-20
0
20

FIGURE 12

Figure 13. Detail of Figure 11 at low amplitudes.

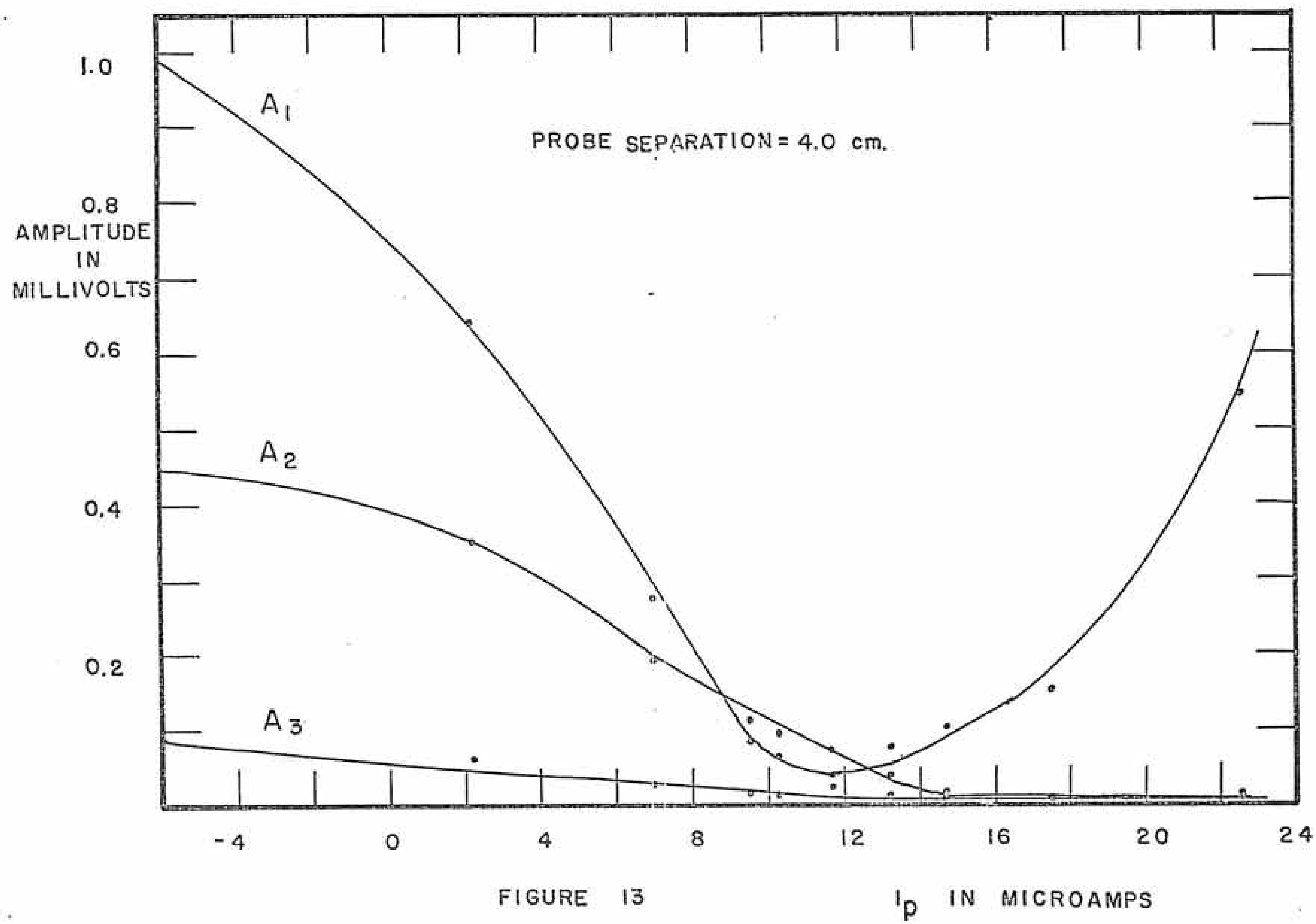

PROBE SEPARATION = 4.0 cm.
AMPLITUDE IN MILLIVOLTS
1.0
0.8
0.6
0.4
0.2
A_1
A_2
A_3
-4
0
4
8
12
16
20
24
I_p IN MICROAMPS

FIGURE 13

Figure 14. Probe separation as a function of negative bias voltage on transmitting probe.

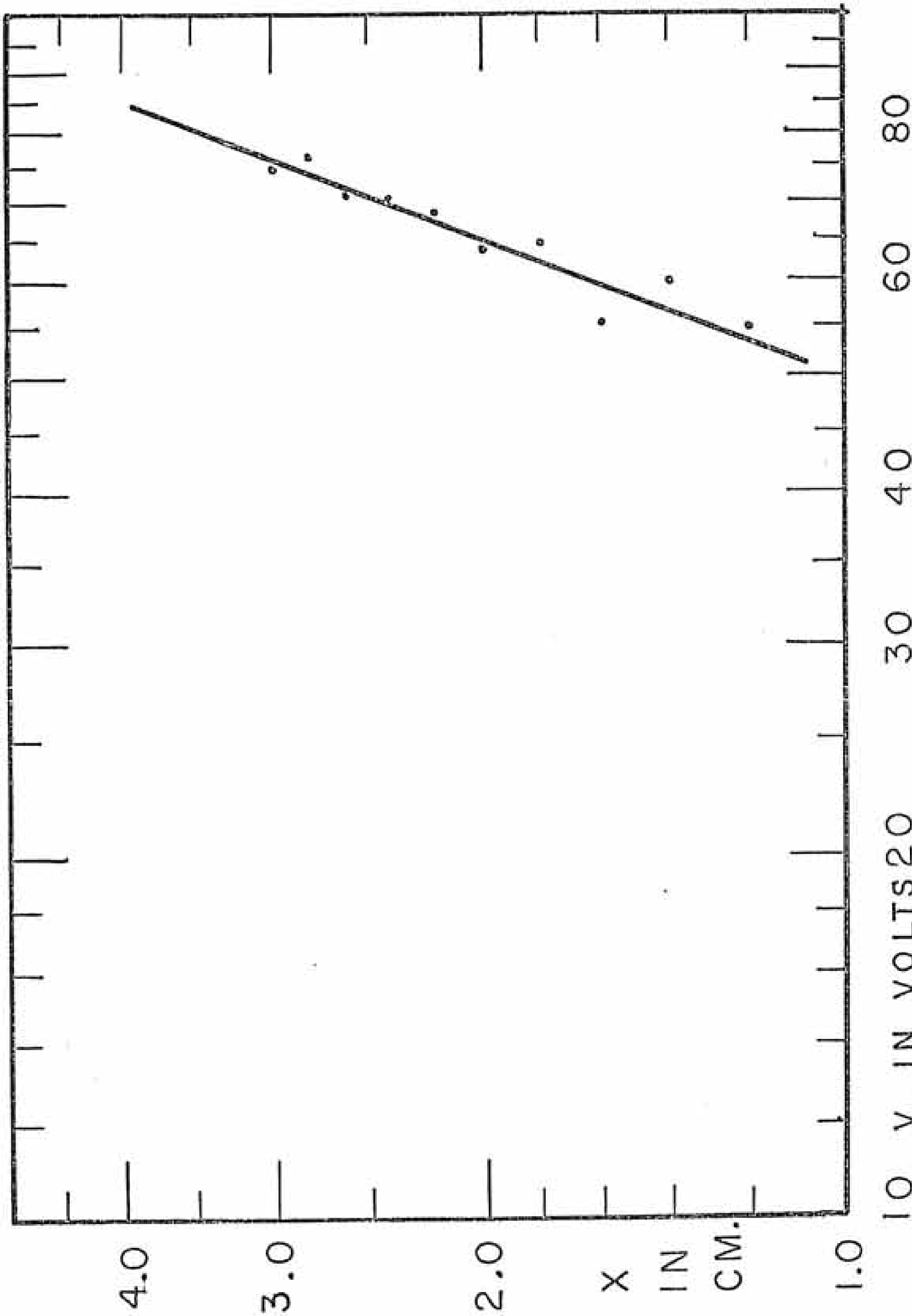


FIGURE 14

Figure 15. Theoretical model of sheath capacity as a function of negative bias voltage compared to response of system at 4.0 centimeters of probe separation.

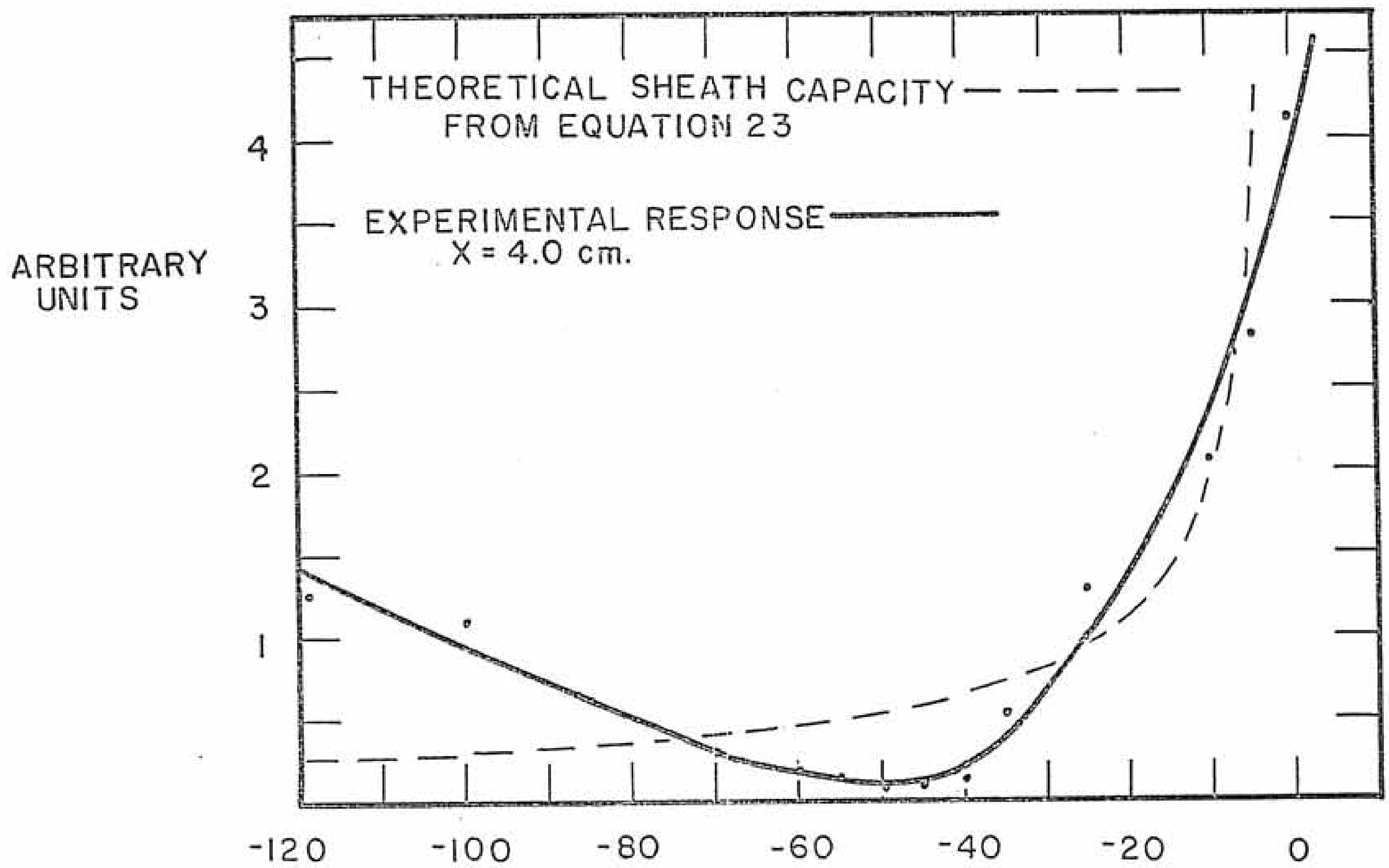

THEORETICAL SHEATH CAPACITY
FROM EQUATION 23
EXPERIMENTAL RESPONSE
X = 4.0 cm.
ARBITRARY
UNITS
4
3
2
1
-120
-100
-80
-60
-40
-20
0
V_p IN VOLTS

FIGURE 15