OBSERVATION OF A WHISTLER IN THE MAGNETOSPHERE OF SATURN

by Ferzan Akalin

A thesis submitted in partial fulfillment of the requirements for the Master of Science degree in Physics in the Graduate College of The University of Iowa

December 2005

Thesis Supervisor: Professor Donald A. Gurnett

Graduate College The University of Iowa Iowa City, Iowa

CERTIFICATE OF APPROVAL

MASTER'S THESIS

This is to certify that the Master's thesis of

Ferzan Akalin

has been approved by the Examining Committee for the thesis requirement for the Master of Science degree in Physics at the December 2005 graduation.

Thesis Committee:

Gurnett, Thesis Supervisor Øonald/A

John A. Goree

a.

Steven R. Spangle

To my mom Nilgun Akalin, to my dad Sener Akalin, to my sister Nilufer Akalin, and to my beloved country Turkey.

ACKNOWLEDGMENTS

I would like to acknowledge my advisor Prof. Donald A. Gurnett for his help at every stage of my study and his tremendous guidance and support. Without him, this study would be impossible to accomplish. This research was supported by NASA through contract 961152 with the Jet Propulsion Laboratory. I would also like to thank Ann Persoon, Terry Averkamp, Bill Kurth, Kathy Kurth, and many others who are involved in this project and who helped me with no hesitation at any stage of my work. Without their contribution this thesis would not be what it is right now. Furthermore, I would like to mention the emotional support of my family in my studies. Even though they live far away from here, their emotional presence and their love were always with me. I would like to thank my mom and my dad for guiding me in the best way possible, my sister for being a great friend, and a huge inspiration for me with her intelligence, understanding, and maturity. Next, I would like to thank my roommates Talia Ekin Tokyay and Birsen Donmez for being great companies and being great friends. Without them Iowa City would definitely be colder. Finally I would like to thank a very special friend for reviewing this thesis for me.

ABSTRACT

This thesis describes and analyzes the first whistler observed in the magnetosphere of Saturn. Whistlers are low-frequency electromagnetic waves produced by lightning. The whistler was detected by the Radio and Plasma Wave Science (RPWS) instrument on the Cassini spacecraft. Cassini was launched on October 15, 1997, and was put in orbit around Saturn on July 1, 2004. During the inbound pass of orbit A at a radial distance of 6.19 R_s (Saturn radii), a signal was detected from about 200 to 400 Hz that had the proper dispersion characteristics to be a whistler. The frequency-time dispersion of the whistler was found to be 81 Hz^{1/2}s. Based on this dispersion constant, we determined, from a travel time computation, that the whistler must have originated from lightning in the northern hemisphere of Saturn. Using a simple centrifugal potential model consisting of water group ions, and hydrogen ions we also determine the fractional concentration and scale height that gave the best fit to the observed dispersion. The fractional concentration of the water group ions was found to be 0.87 with a scale height of 1.08 R_s (Saturn radii), and the fractional concentration of the hydrogen group ions was found to be 0.13 with a scale height of 4.32 R_s. In addition to the centrifugal potential model, a more complex diffusive equilibrium model was also analyzed, where the plasma consists of electrons, hydrogen ions, and singly charged oxygen ions. The fractional concentration of the oxygen ions in this model was found to be 0.85, and the fractional concentration for the protons was found to be 0.15. The inferred electron density profiles for these two models are very similar.

TABLE OF CONTENTS

LIST OF FIGURES		
CHAPTER I	INTRODUCTION	1
CHAPTER II	OBSERVATIONS	3
CHAPTER III	DISPERSION ANALYSIS	4
3.1 3.2 3.3 3.4	Dispersion Relation A Simple Density Model A Simple Multispecies Model An Ambipolar Equilibrium Density Model	
CHAPTER IV	CONCLUSION	12
APPENDIX A FIGURES		
APPENDIX B NUMBER DENSITY MODEL		
REFERENCES		

LIST OF FIGURES

Figure A1.	A frequency-time spectrogram showing the strongest and clearest whistler detected. Orbital parameters are radial distance in Saturn radii, magnetic latitude in degrees, and magnetic local time in hours. The frequency range of the spectrogram for this whistler is 100 to 600 Hz, and the time range is 15 seconds
Figure A2.	Frequency-time spectrogram showing the two whistler-like events detected. Orbital parameters are radial distance in Saturn radii, magnetic latitude in degrees, and magnetic local time in hours. The frequency range for the spectrum is 0-20 kHz, and the time range is 30 seconds
Figure A3.	Inverse frequency values of the whistler are plotted as a function of time. The slope of the plot gives 1/D17
Figure A4.	Illustration of how the whistlers are produced and how they follow the magnetic field line to reach the spacecraft
Figure A5.	Dispersion constant is plotted as a function of scale height both for northern and southern hemisphere sources, and the observed $D=81$ $Hz^{1/2}sec$ only corresponds to a scale height value in northern hemisphere case
Figure A6.	Number density is plotted as a function of arc length to show the concentration fraction of hydrogen group and water group ions of plasma along the field line
Figure A7.	Frequency vs. time plot of the actual whistler and the best fit that was obtained for the whistler
Figure A8.	Optimized density models which fit the observed whistler dispersion. The plasma density is plotted as a function of arc length measured along the field line for two models. Solid line in the plot shows the results from the model from equation (7). The dashed line shows the model from equation (10). This model was provided by Ondrej Santolik, and the code that analyses the model from equation (10) is called "fitcas.pro"

CHAPTER I INTRODUCTION

Whistlers are low-frequency electromagnetic waves produced by lightning. They propagate along the magnetic field line of a planet at frequencies below the electron cyclotron frequency and the electron plasma frequency. Whistlers were discovered in 1918 by Barkhausen [1919] using an antenna connected to a rudimentary vacuum tube amplifier. The signals that he detected consisted of audio frequency tones that decreased in frequency with increasing time, hence the term whistlers. He thought that whistlers were somehow related to meteorological effects. Later, Eckerslev [1935] carried out a detailed study of whistlers and showed that the time delay varied as D/\sqrt{f} , where D is a constant called the dispersion and f is the frequency. This relationship is called the Eckersley law. He also suggested that the signals were produced by lightning, but he could not explain the long delay time, which was sometimes several seconds or more. Storey [1953] was the first to confirm that whistlers were produced by lightning. He observed a lightning flash, and heard the whistling tone a few seconds later. Storey was also the first to propose the correct theoretical explanation of whistlers. He showed that whistlers propagate along the Earth's magnetic field lines from one hemisphere to the other in a plasma mode of propagation that we now call the whistler mode. We also assumed parallel propagation in the analysis of the whistler, because the group velocity direction relative to the magnetic field is always less than 19⁰28" for whistler mode waves. At the very low frequencies where whistlers occur, it can be shown that the higher frequencies propagate faster than the lower frequencies. The impulsive signal produced by a lightning flash is thereby converted into a whistling tone as it propagates along the magnetic field line, with the highest frequencies arriving first. Since then, whistlers have been detected at two other planets. Whistlers were observed at Jupiter in 1979 during the Voyager 1 flyby of the giant planet [*Scarf et al.*, 1979; *Gurnett et al.*, 1979]. Whistler dispersion measurements were later used to put constraints on the light ion distribution in the inner region of the Jovian magnetosphere [*Tokar et al.*, 1982]. A search for whistlers was conducted during both the Voyager 1 and Voyager 2 flybys of Saturn, but none were found, possibly because of the very limited amount of data obtained during these flybys. No whistlers were detected during the Voyager 2 flyby of Uranus in 1986. However, whistlers were detected during the Voyager 2 flyby of Neptune in 1989 [*Gurnett et al.*, 1990].

In this thesis we report the first whistler to be observed in the magnetosphere of Saturn. This whistler was observed by the Radio and Plasma Wave Science (RPWS) instrument on the Cassini spacecraft. Cassini was put in orbit around Saturn on July 1, 2004. For a description of the Cassini mission to Saturn see *Matson et al.* [2002], and for a description of the RPWS instrument see *Gurnett et al.* [2004].

CHAPTER II OBSERVATIONS

The first and so far the only event detected at Saturn that we believe is a whistler occurred on October 28, 2004. A frequency-time spectrogram of this event is shown in Figure 1. This event is observed at 09:58:53 UT (Universal Time) during the inbound pass of orbit A. At that time the spacecraft was north of the equatorial plane at a latitude of 12.31 degrees, a radial distance of 6.18 R_s , and a local time of 18.47 hours. The duration of the whistler was about 3 seconds and the frequency varied from 430 to 200 Hz. Because the group velocity of whistler mode waves is greater at higher frequencies than at lower frequencies, the higher frequencies are detected first. This property is called dispersion. Since the frequencies are within the audible range, they produce a whistling tone when they are converted to sound.

Figure 2 shows a frequency-time spectrogram of two additional whistler-like events that were observed in the magnetosphere of Saturn on July 1, 2004. These events took place at 4:01:23 and 04:01:26 UT (Universal Time). At this time Cassini was at a radial distance of about 2.15 R_s and a latitude of 3.94 degrees, just north of the ring plane. The time between the two whistler-like events is about 2 seconds, and the total duration from the beginning of the first whistler-like event to the end of the second event is about 5 seconds. The analysis of the dispersion of these two events did not give a good fit to Eckersley law for whistlers, so we are doubtful that they are whistlers. The signals also have a very poor signal to noise ratio. Because of these reasons they will not be analyzed any further in this thesis.

CHAPTER III DISPERSION ANALYSIS

3.1 Dispersion Relation

The dispersion of a whistler can usually be described by Eckersley law [*Eckersley*, 1935], which is given by

$$t = t_0 + D/\sqrt{f} \quad , \tag{1}$$

where t is the arrival time of the whistler at frequency f, t_0 is the time of the lightning flash, and the quantity D is a constant called the dispersion. To compute the dispersion of the whistler in Figure 1, the arrival time and the frequency of the whistler have been manually digitized from the spectrogram. A plot of $1/\sqrt{f}$ versus t was then constructed to determine whether the observed arrival time fits Eckersley law. If Eckersley law is obeyed, the measured points should be in a straight line. Such a plot is shown in Figure 3. As can be seen, the measured arrival times provide a good fit to a straight line. The dispersion constant D is given by inverse of the slope of the line and is $D = 81 \text{ Hz}^{1/2} \text{sec.}$

Since whistlers are known to propagate along a magnetic field line [*Storey*, 1953], the propagation path for the whistler detected at Saturn is as shown in Figure 4, assuming for the moment that the lightning source is in the northern hemisphere. The dispersion constant is determined by the plasma density distribution along the magnetic field line and can be calculated using the following equation where the integration is carried along the magnetic field line from the lightning source to the spacecraft [*Helliwell*, 1965]

$$D = \frac{1}{2c} \int \frac{f_{p}}{\sqrt{f_{c}}} ds \quad .$$
 (2)

The frequency $f_p = 8980\sqrt{n_e}$ Hz is the electron plasma frequency, where n_e is the electron density in cm⁻³, and the frequency $f_c = 28B$ is the electron cyclotron frequency, where B is the magnetic field strength in nT. Equation (2) is valid if the wave frequency is much less than both the electron cyclotron frequency (f « f_c) and the electron plasma frequency (f « f_p). The plasma density must also satisfy (ff_c« f_p^2). Since the axis of Saturn's dipole magnetic field is aligned within one degree of its rotational axis, magnetic field strength at a given point along the magnetic field line is given to a good approximation by the following equation [*Parks*, 1991]

$$B = B_0 \frac{\sqrt{1 + \sin^2 \lambda}}{\cos^6 \lambda} \quad , \tag{3}$$

where B_0 is the surface magnetic field at the equator, and λ is the latitude. The magnetic field strength measured at the spacecraft was provided by the magnetometer of Cassini and is B(measured)=89.513 nT. By using the measured magnetic field strength at the spacecraft in equation (3), B_0 can be evaluated and is B_0 =76.1 nT.

The path length along the magnetic field line, s, can be evaluated using [*Helliwell*, 1965]

$$s = \frac{1}{2} \frac{R_s}{\sqrt{3} \cos^2 \lambda_0} \left(\chi + \sinh \chi \cdot \cosh \chi \right) , \qquad (4)$$

where R_s is the mean radius of Saturn (60,268 km), λ_0 is the latitude of field line at R_s , and $\chi = \log(\sqrt{3}\sin\lambda + \sqrt{3(\sin^2\lambda + 1)})$, where λ is the latitude [*Helliwell*, 1965]. Since the whistler occurred in the atmosphere of Saturn, it is useful to know the latitude at the foot of the magnetic field line (i.e., r=R₀). This latitude can be found using the equation [*Kivelson et al.*, 1995]

$$\lambda_0 = \cos^{-1}(\sqrt{1/R_0}) \tag{5}$$

which for R₀=6.49 R_s, λ_0 is 66.9⁰ where R₀ can be calculated from $r = R_0 \cos^2 \lambda$ [*Helliwell*, 1965].

3.2 A Simple Density Model

The objective of this part of the thesis is to make an initial calculation of the dispersion using equation (2) and a simple model for the plasma density distribution along the magnetic field line. To compute the integral in equation (2), n_e must be known as a function of latitude in order to evaluate the electron plasma frequency. The plasma in Saturn's magnetosphere beyond the rings is known to be rotating as though it is rigidly locked to Saturn's rotation. This rotation produces a centrifugal force that causes the plasma to accumulate in a disk near the equatorial plane. This disk is called the plasma disk (see Figure 4). If the plasma consist of a single species of ions and an equal density of electrons the number density, n_e , in the plasma disk can be obtained from the following equation,

$$n_{e} = n_{0} \exp\left[-\frac{1}{3} \frac{R_{0}^{2}}{H^{2}} (1 - \cos^{6} \lambda)\right], \qquad (6)$$

where n_0 is the number density at the equator, R_0 is the equatorial radius of the magnetic field line, λ is the geomagnetic latitude and H is a quantity called the scale height. Equation (6), which is derived in the Appendix B, originates from a principle of equilibrium statistical mechanics that states that the density is proportional to $\exp[-W/kT]$ where W is the potential energy, k is Boltzmann's constant and T is the temperature. The potential energy in this case is due to the centrifugal force. In this model gravitational force is neglected because, at radial distances beyond about 2 R_s, the gravitational force is small compared to the centrifugal force. The scale height provides a measure of the north-south thickness of the plasma disk, and as shown in the Appendix B is given by $H = \sqrt{2kT/3m\Omega^2}$, where m is the ion mass and Ω is the rotation rate of Saturn.

To see if this simple model can account for the observed dispersion, the dispersion given by equation (2) was computed using equation (6) for scale heights ranging from 1 R_s to 4 R_s using Newton's method. Scale heights outside of this range would not be consistent with previous plasma measurements at Saturn [*Richardson*, 1995]. The results are shown by the plot in Figure 5. This plot shows that for lightning in the northern hemisphere the dispersion ranges from around 50 to 120 Hz^{1/2}s. For the lightning in the southern hemisphere the dispersion ranges from about 420 to 240 Hz^{1/2}s. Since the measured dispersion constant is 81 Hz^{1/2}s, only a northern hemisphere source matches the observed dispersion of the whistler. Therefore, we conclude that the lightning source must have been in the northern hemisphere. The best fit scale height that matches the observed dispersion is $H = 2.2 R_s$.

3.3 A Simple Multispecies Model

Plasma measurements on the Cassini spacecraft show that the plasma disk consists of hydrogen ions (protons) and water group ions [*Young et al.*, 2005]. To provide a better model of the plasma density distribution we have generalized equation (2) to include two species.

$$n_{e} = \sum_{i=1}^{2} n_{i} = n_{0} \sum_{i=1}^{2} \alpha_{i} \exp\left[-\frac{1}{3} \frac{R_{o}^{2}}{H_{i}^{2}} (1 - \cos^{6} \lambda)\right]$$

$$\alpha_{2} = 1 - \alpha_{1}$$
(7)

In the above equation α is the fractional concentration of each ion species. Since the water group ions are roughly 16 times as massive as hydrogen ions, H₂ is assumed to be 4H₁, where H₂ is the scale height for the hydrogen ions, and H₁ is the scale height for the water group ions. Equation (2) that was used in section 3.1 fails for high latitudes, because number density gets smaller as the magnetic latitude increases, eventually, violating the condition (ff_c«f_p²). For this reason we use a more precise model for the group index of refraction which is given by

$$n_{g} = \frac{1 + (1/2) \left[f_{p}^{2} f_{c} / f (f_{c} - f)^{2} \right]}{\left[1 + f_{p}^{2} / f (f_{c} - f) \right]^{1/2}}$$
(8)

where $n_{g,}$, the group index of refraction depends on the electron plasma frequency and electron cyclotron frequency [*Gurnett et al.*, 1979]. The group index of refraction in equation (8) is then used to determine the time delay of the whistler using the equation

$$t = \frac{1}{c} \int n_{g} ds .$$
 (9)

To provide the best fit to the observed dispersion, both the scale height and the fractional concentrations of the species had to be varied since they are both unknowns in

equation (7). For each value of number density that was obtained from equation (7), the plasma frequency was derived.

Doing the same analysis for every point along the magnetic field line, a minimum chi squared was obtained when the scale height for the water group ions was 1.08 R_s, with a fractional concentration of 0.84 at the equator, and the scale height for the hydrogen group ions was 4.32 R_s, with a fractional concentration of 0.16 at the equator. Figure 6 shows the number density profile for the hydrogen ion and water group ion species. Figure 7 shows the time delay fit for the whistler where the total number density profile is provided by Ann Persoon [*Persoon et al.*, 2005].

3.4 An Ambipolar Equilibrium Density Model

For an even more detailed approach, an ambipolar equilibrium model was evaluated. In this approach, we include the ambipolar electric field force that was neglected in the previous model. The ambipolar electric field is a consequence of the fact that electrons have a much larger thermal speed than ions. The electrons thus escape from a potential minimum much more easily than ions. In this case the potential minimum is created by the centrifugal force acting on particles which can only move along the field lines. This creates an electric field directed outwards from the minimum potential which prevents electrons from escaping and separating from the ions. This electric field is called the ambipolar electric field. As in the previous model we will use two ion species, hydrogen ions and singly charged oxygen ions to represent the water group ions. The plasma density is then calculated using the following equation for each species [*Richardson*, 1995, 1998]:

$$\frac{\partial \mathbf{P}_{\parallel}}{\partial s} - \left(\mathbf{P}_{\parallel} - \mathbf{P}_{\perp}\right) \frac{1}{B} \frac{\partial \mathbf{B}}{\partial s} - \mathbf{n}_{i} \mathbf{m}_{i} \frac{\partial}{\partial s} \left(\frac{1}{2}\Omega^{2}\rho^{2}\right) + \mathbf{n}_{i} \frac{\partial}{\partial s} \left(\frac{\mathbf{G}\mathbf{M}_{s}\mathbf{m}_{i}}{\mathbf{r}}\right) + \mathbf{n}_{i} \mathbf{Z}_{i} \mathbf{q} \frac{\partial \phi}{\partial s} = 0.$$
 (10)

Here, the first term is the pressure gradient force. The second term is force due to the magnetic mirror effect. This term is equal to zero since we assume plasma that is isotropic, i.e. $P_{\parallel} = P_{\perp}$. The third term is the centrifugal force where ρ is the distance from Saturn's rotational axis, Ω is the angular velocity of Saturn, m_i is the mass of each ion species i, and n_i is the number density of each ion species i. The fourth term in the equation is the gravitational force, where G is the gravitational constant, and M_s is the mass of Saturn. The gravitational force term is only important close to the planet, otherwise it gets small compared to the centrifugal force. The last term of the equation is the force induced by the ambipolar electric field between different species in the plasma with different masses and electric charges. This equation should be written for each species in the plasma using the charge neutrality in order for the equation to be closed.

The boundary conditions for this model are measurements of the plasma density onboard the Cassini spacecraft. We use an offset dipole model of the Saturn's magnetic field [*Connerney et al.*, 1982]. The ion composition at the magnetic equator is a free parameter of the model. The electron temperature is fixed at 1 eV for a given ion composition and the ion temperature is calculated using the iterative nonlinear Newton's method to fit the observed density at the spacecraft position and at the magnetic equator.

The fractional concentration for hydrogen ions which best corresponds to the observed whistler dispersion is 0.15 at the equator, and the fractional concentration for oxygen ions is 0.85 at the equator, both evaluated at the equator. The corresponding ion temperature is 19.7 eV. Figure 8 demonstrates that the models shown in this section and

in the previous section are very similar along the magnetic field line close to the equator. The difference in the number densities of the two models increases as the latitude increases as shown in Figure 8. This difference is mainly due to the gravitational force which was not included in equation (7).

CHAPTER IV CONCLUSION

Evidence for the existence of whistlers in Saturn's magnetosphere has been presented in this thesis. Even though three whistler-like events were found, only the whistler that is examined in this thesis can be convincingly described as a whistler. Because only a northern hemisphere source gives an acceptable dispersion the lightning responsible for this whistler is believed to have occurred in the northern hemisphere at a latitude of about 67 degrees.

Saturn electrostatic discharges (SEDs), which are believed to originate from lightning, are commonly observed at Saturn [*Kaiser et al.*, 1984]. Both SEDs and whistlers are caused by lightning flashes. During an atmospheric storm observed by the Cassini imaging system at 35° south latitude [*Porco et al.*, 2005], the RPWS instrument detected SEDs corresponding to convective could features [*Gurnett et al.*, 2005]. The storm system responsible for these SEDs was in the southern hemisphere of Saturn, whereas the whistler that we observed occurred in the northern hemisphere. The latitude at which the whistler originated is surprising since the atmosphere is in continuous darkness, whereas the lightning responsible for the SEDs was observed in the sunlight hemisphere. Most likely the lightning responsible for the whistler was driven by an internal heat source rather than by sunlight. We also looked for SEDs at the time when the whistler was observed, but none were found.

The whistler dispersion gives us several important constraints on the density and temperature of the equatorial plasma disk that exists at Saturn. The plasma in Saturn's inner magnetosphere is believed to consist of two main group ions: Water group ions, and hydrogen group ions. The whistler dispersion measurements show that the concentration fraction of the water group ions is 0.86 at the equator, with a scale height of 1.08 Saturn radii, and that the concentration fraction of the hydrogen group ions is 0.14 at the equator, with a scale height of 4.32 Saturn radii. This scale height corresponds to an ion temperature of about 19.7 eV.

A more complicated model that includes a force term due to the ambipolar electric field was also considered. The results from both of the models are in very close agreement. In other words, for the electron and ion temperatures that exist in Saturn's plasma disk the ambipolar electric field does not have a big effect.

APPENDIX A FIGURES



Figure A1. A frequency-time spectrogram showing the strongest and clearest whistler detected. Orbital parameters are radial distance in Saturn radii, magnetic latitude in degrees, and magnetic local time in hours. The frequency range of the spectrogram for this whistler is 100 to 600 Hz, and the time range is 15 seconds.



Figure A2. Frequency-time spectrogram showing the two whistler-like events detected. Orbital parameters are radial distance in Saturn radii, magnetic latitude in degrees, and magnetic local time in hours. The frequency range for the spectrum is 0-20 kHz, and the time range is 30 seconds.



Figure A3. Inverse frequency values of the whistler are plotted as a function of time. The slope of the plot gives 1/D.



Figure A4. Illustration of how the whistlers are produced and how they follow the magnetic field line to reach the spacecraft.



Figure A5. Dispersion constant is plotted as a function of scale height both for northern and southern hemisphere sources, and the observed D = 81 Hz^{1/2}sec only corresponds to a scale height value in northern hemisphere case.



Figure A6. Number density is plotted as a function of arc length to show the concentration fraction of hydrogen group and water group ions of plasma along the field line.



Figure A7. Frequency vs. time plot of the actual whistler and the best fit that was obtained for the whistler.



Figure A8. Optimized density models which fit the observed whistler dispersion. The plasma density is plotted as a function of arc length measured along the field line for two models. Solid line in the plot shows the results from the model from equation (7). The dashed line shows the model from equation (10). This model was provided by Ondrej Santolik, and the code that analyses the model from equation (10) is called "fitcas.pro".

APPENDIX B NUMBER DENSITY MODEL

Here centrifugal force is defined as:

$$\mathbf{F} = \mathbf{m}\Omega^2 \boldsymbol{\rho} \tag{B1}$$

where F is centrifugal force, m is the mass of each ion species, Ω is the angular velocity of the plasma, and $\rho = r \cos \lambda = R_0 \cos^3 \lambda$ is the distance from spin axis where $r = R_0 \cos^2 \lambda$.

Potential can be derived from force as shown in the next equation.

$$W = -\int F_{e} d\rho = -\frac{1}{2} m \Omega^{2} \rho^{2} = -\frac{1}{2} m \Omega^{2} R_{0}^{2} \cos^{6} \lambda$$
 (B2)

Number density profile from statistical mechanics is:

$$\mathbf{n}_{i} = \mathbf{n}_{0} \exp\left[-\frac{W}{kT}\right] = \mathbf{n}_{e} \exp\left[-\frac{m\Omega^{2}R_{0}^{2}}{2kT}\cos^{6}\lambda\right]$$
(B3)

Next, we subtract the potential at $\lambda = 0$. Equation (A3) becomes:

$$W = -\frac{1}{2} m \Omega^{2} (R_{0}^{2} - \rho^{2})$$

= $-\frac{1}{2} m \Omega^{2} (R_{0}^{2} - R_{0}^{2} \cos^{6} \lambda)$
= $-\frac{1}{2} m \Omega^{2} R_{0}^{2} (1 - \cos^{6} \lambda)$ (B4)

Scale height is defined as $H = \sqrt{2kT/3m\Omega^2}$. It can easily be shown that $kT = 3m\Omega^2 H^2/2$. When kT, and equation (A4) are substituted into equation (A3), we achieve

$$n_{e} = n_{0} \exp\left[-\frac{m\Omega^{2}R_{0}^{2}}{3H^{2}}(1-\cos^{6}\lambda)\right].$$
 (B5)

REFERENCES

- Barkhausen, H., Zwei mit Hilfe der neuen Verstarker entdeckte Erscheinungen, *Phys. Z.*, 20, 401, 1919.
- Connerney, J.E.P., et al., Zonal Harmonic Model of Saturn's Magnetic Field From Voyager 1 and Voyager 2 Observations, *Nature*, 298, 5849, 1982.
- Eckersley, T.L., Musical Atmospherics, Nature, 135, 104, 1935.
- Gurnett, D.A., R.R. Shaw, R.R. Anderson, and W.S. Kurth, Whistlers Observed by Voyager 1: Detection of Lightning on Jupiter, *Geophys. Res. Lett.*, 6, 6, 1979
- Gurnett, D.A., W.S. Kurth, I.H. Cairns, and L.J. Granroth, Whistlers in Neptune's Magnetosphere: Evidence of Atmospheric Lightning, *J. Geophys. Res.*, 95, A12, 1990.
- Gurnett, D.A., et al. The Cassini Radio and Plama Wave Investigation, *Space Sci. Rev.*, *114*, 395-463, 2004.
- Gurnett, D.A., et al., Radio and Plasma Wave Observations at Saturn from Cassini's Approach and First Orbit, *Science*, *307*, 5713, 2005.
- Helliwell, R.A., *Whistlers and Related Ionospheric Phenomena*, Stanford University Press, Stanford, Calif., 1965
- Kaiser, M.L., et al., Saturn's Ionosphere: Inferred Electron Densities, J. Geophys. Res., 89, A4, 1984.
- Kivelson, Margaret G., et al., *Introduction to Space Physics*, Cambridge University Press, New York, NY, 1995
- Matson, D.L., L.J. Spilker, and J.P. Lebreton, The Cassini/Huygens Mission to The Saturnian System, *Space Sci. Rev.*, 104, 1-58, 2002.
- Parco, C.C., et al., Cassini Imaging Science: Initial Results on Saturn's Atmosphere, *Science*, 307, 5713, 2005.
- Parks, George, K. *Physics of Space Plasmas, An Introduction*, Addison-Wesley Publishing Company, Redwood City, Calif, 1991.
- Persoon, A.M., et al., Equatorial Electron Density Measurements in Saturn's Inner Magnetosphere, *Geophys. Res. Lett.*, in press by November 2005.
- Richardson, J.D., An Extended Plasma Model for Saturn, Geophys. Res. Lett, 22, 10, 1995.

- Richardson, J.D., Thermal Plasma and Neutral Gas in Saturn's Magnetosphere, *Reviews of Geophysics*, *36*, 4, 1998.
- Storey, L.R.O., An Investigation of Whistling Atmospherics, *Philos. Trans. R. Soc. London, Ser.* A. 246, 113, 1953.
- Tokar, R.L., D.A Gurnett, F. Bagenal, and R.R. Shaw, Light Ion Concentrations in Jupiter's Inner Magnetosphere, J. Geophys. Res., 87, A4, 1982.
- Young, D.T., et al., Composition and Dynamics of Plasma in Saturn's Magnetosphere, *Science*, 307, 2005.