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Formation of VLF chorus frequency spectrum: Cluster data and comparison with the backward wave oscillator model

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[1] The dependence of the frequency spectrum of individual chorus elements on the position of the observation point in and near the generation region is analyzed using recent Cluster data obtained on two different geomagnetically active days. The source of night-side chorus is localized using multicomponent measurements of the wave electric and magnetic fields. We have revealed that the spectrum of the chorus elements lacks the lower frequencies at the center of the source region. One possible explanation of this effect is provided by applying the backward wave oscillator model of chorus generation to these data. According to this model, the chorus frequency is determined by the parallel velocity corresponding to a step-like deformation in the distribution function of resonant electrons. This velocity decreases during the generation of an element as the electrons move through the source region. Thus, only a part of a chorus element is visible inside this region. For the typical case of rising-tone chorus elements, the lower frequencies are generated downstream with respect to the chorus propagation and, hence, disappear as a receiver is moved upstream towards the center of the source region. **Citation:** Trakhtengerts, V. Y., A. G. Demekhov, E. E. Titova, B. V. Kozelov, O. Santolik, E. Macusova, D. Gurnett, J. S. Pickett, M. J. Rycroft, and D. Nunn (2007), Formation of VLF chorus frequency spectrum: Cluster data and comparison with the backward wave oscillator model, *Geophys. Res. Lett.*, *34*, L02104, doi:10.1029/2006GL027953.

1. Introduction

[2] Recent multi-satellite Cluster data [*Parrot et al.*, 2003; *Santolik et al.*, 2003, 2004] permit a detailed investigation of the generation region of chorus emissions. According to this data, the chorus source is localized near the equatorial cross-section of a magnetic flux tube and has typical scale ∼ 2000 km along the magnetic field [see also *LeDocq et al.*, 1998]. The wave energy flux S travels in both directions inside this region and is directed away from the equator outside it.

[3] In this paper we continue analysis of Cluster data in order to investigate the dependence of the frequency spectrum of individual chorus elements on the position of observation point in and near the generation region. This information is very important for elucidating the generation mechanism of chorus emissions. We show that the frequency-space dependence following from our analysis of Cluster data permits us to make an important conclusion concerning the formation of the chorus frequency spectrum. We also discuss a simple theoretical model, based on the backward-wave oscillator (BWO) mechanism [*Trakhtengerts*, 1999; *Trakhtengerts et al.*, 2004], which can explain the observed features of the chorus spectra.

[4] In our analysis, we study how the frequency range occupied by an individual chorus element depends on the position of the detector with respect to the generation region. For that we have chosen two specific examples of Cluster VLF data obtained on geomagnetically highly disturbed days of 18 April 2002 and 31 March 2001, when intense chorus emissions were observed on the nightside [*Santolik et al.*, 2003, 2004].

2. Cluster Data on Chorus Spectrum

[5] The intense whistler mode chorus emissions for both events were detected in the premidnight sector in the generation region close to the magnetic equatorial plane at a radial distance of about 4 Earth radii during strong geomagnetic disturbances. In past work, the position of the source region was obtained by using multicomponent measurements of the electric and magnetic fields from the spectrum analyzers of the STAFF instrument [*Cornilleau-Wehrlin et al.*, 1997]. The overview spectrograms and detailed analysis of these events are given by [*Santolik et al.*, 2003, 2004].

[6] Inside and near the source region, we analyzed the lower band of chorus below one half of the electron cyclotron frequency f_{ce}, obtained from high resolution data of the wideband (WBD) plasma wave instrument [*Gurnett et al.*, 2001]. We determined the lower and upper frequencies of each chorus element for both events and normalized them to the local gyrofrequency f_{ce}. Altogether, about 3000 discrete elements were identified on spectrograms from the WBD instrument. The normalized lower and upper frequen-

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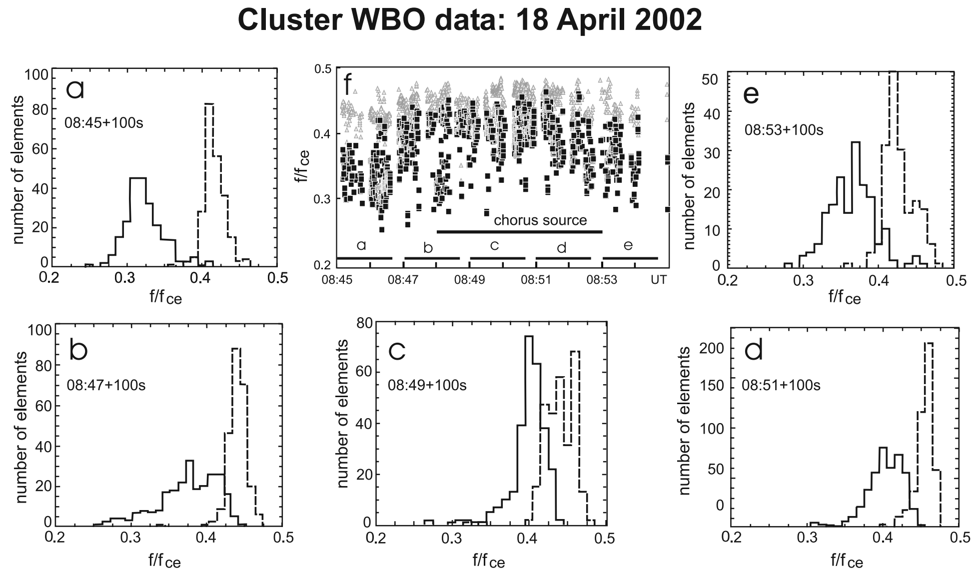

Figure 1. (a, b, c, d, and e) The distributions of normalized lower and upper frequencies of chorus elements, obtained for 100-s intervals for different Cluster 1 locations with respect to the chorus source, are shown by solid and dashed lines, respectively. Start time of the 100-s interval is indicated. (f) The normalized lower (black) and upper (gray) frequencies of chorus elements detected by Cluster 1 spacecraft on April 18, 2002. The chorus source region obtained from Poynting flux measurements is marked by a horizontal line. The horizontal bars labeled a to e indicate the corresponding time intervals.

cies of the chorus elements detected by the Cluster 1 spacecraft on April 18, 2002 are shown in Figure 1f. The region in which the parallel component of the Poynting flux is close to its standard deviation is marked by a horizontal line from 08:48 to 08:53 UT on Figure 1f. This region can be approximately identified as the chorus source region, with a note that the boundaries of the actual source are not sharp, and such a definition is not strict. If we define the source region as the region in which the electromagnetic planarity factor is close to zero [see *Santolik et al.*, 2004], then the source is slightly larger than is shown in Figure 1. Figures 1a, 1b, 1c, 1d, and 1e demonstrate 100-s distributions of the lower and upper frequencies of chorus elements for different Cluster 1 positions with respect to the chorus source. A difference between low frequencies of chorus elements observed in the middle of the source and close to its boundaries (i.e., a frequency increase in lower frequencies near the source center) is clearly seen in Figure 1f. The upper frequencies also increase slightly but this increase is significantly smaller. This conclusion is supported by interval-based statistics presented in Figures 1a–1e. Outside the source or further out from its center, the distributions of lower and upper frequencies are well separated, while they are statistically close to each other near the source center. The lower frequencies appear again as the spacecraft moves to the opposite side of the source region.

[7] The observed lack of lower chorus frequencies in the center of the source is difficult to explain by propagation effects, since propagation away from a source on the same field line would rather lead to disappearance of some spectral components (e.g., due to refraction) further out from the source center, than to the appearance of new ones. On the other hand, if lower frequencies of chorus elements would come from other field lines with weaker magnetic field, then in such a case the upper frequencies of the same elements would have changed proportionally to the lower frequencies, which is not the case in the example that we have presented in Figure 1.

[8] Note that well defined chorus elements suitable for our analysis were absent from 08:55 and appeared again at about 08:58, when the distance of the spacecraft to the dipole magnetic equator rose to 4.5°. We exclude this later time interval from consideration to avoid the influence of propagation effects. It seems to us most feasible to explain our observation by assuming that the lower frequencies of an ensemble of chorus elements are generated closer to the source boundaries, while their upper frequencies are generated over the entire source. In the next section, we show that such an assumption agrees with the picture of chorus generation following from the BWO model.

[9] Similar but less pronounced results have been obtained for the case of March 31, 2001. This case is characterized by rather large motions of the source region [*Santolik et al.*, 2004], so the direction of the Poynting vector was variable. Because of these variations, it was difficult to group elements for statistical analysis according to certain spacecraft position with respect to the source. However, an analysis of Figure 1b in [*Santolik et al.*, 2004] reproduced here as Figure 2 allows us to find additional support for our scheme of chorus spectrum formation. Figure 2 shows that at lower frequencies, the Poynting flux direction was well determined, being either positive or negative, while at higher frequencies, the magnitude of Poynting vector was close to its standard deviation. Thus, the higher-frequency parts of chorus elements propagated in both directions at the same place inside the chorus source, and lower-frequency parts had a predominant propagation direction. This was true for the period from 07:05 to 07:15 UT, when the spacecraft was close or inside the (oscillating) source region. Before and after that period, when the spacecraft was well outside the source, all parts of chorus spectrum had a definite direction of the Poynting

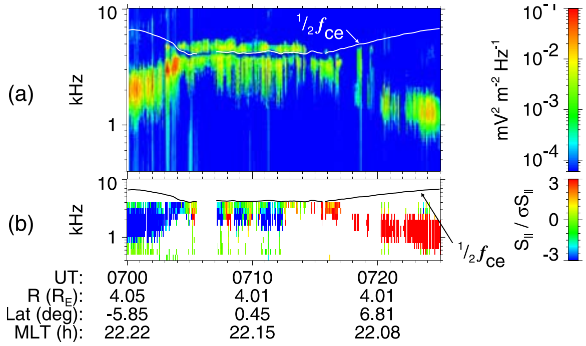

Figure 2. Parallel Poynting flux normalized to its standard deviation, as measured along Cluster 3 orbit on March 31, 2001 [from *Santolik et al.*, 2004].

flux from the source. This result is consistent with lower frequency observations in the time interval 07:05 to 07:15 being due to emissions generated closer to the downstream boundaries of the source region.

3. Scheme of Chorus Spectrum Formation

[10] We shall discuss these results on the basis of the backward wave oscillator model of chorus generation [*Trakhtengerts*, 1999]. This model relates this process to an absolute instability of whistler-mode waves which can occur in the presence of a step-like feature in the distribution function of energetic electrons in the parallel velocity components with respect to the magnetic field. Such a step-like deformation can arise naturally due to the cyclotron interaction of electrons with noise-like emissions [*Trakhtengerts et al.*, 1986, 1996; *Nunn and Sazhin*, 1991].

[11] According to the backward-wave oscillator model, chorus generation takes place in a rather small region ($l \approx$ 2000 km) centered near the equatorial cross-section of a flux tube. Electrons having a step-like distribution move along the geomagnetic field in $\pm z$ directions with the velocity $\pm v_{\rm step}$ and independently generate chorus waves propagating in opposite directions (i.e., with group velocities $\mp v_{\rm gr}$, respectively). The BWO theory can explain basic chorus parameters such as chorus growth rate, amplitude, frequency drift rate, and time intervals between the elements, as discussed by [*Titova et al.*, 2003] and [*Trakhtengerts et al.*, 2004]. This supports the hypothesis of the step formation, while the step feature was not directly observed experimentally. The lack of step observations can be related to the transient nature of this feature and also to the too high pitch-angle and energy resolutions (about 1%) which are necessary for its detection and are inaccessible even by modern particle detectors (see also a discussion in the work of *Trakhtengerts et al.* [2001]).

[12] A frequency $\omega(z_g)$ of chorus generated at a point with coordinate z_g along the ambient magnetic field $\vec{B}$ is determined by the cyclotron-resonance condition

$$\omega_{ce}(z_g) - \omega(z_g) = |k_{\parallel}| v_{\rm step}, \tag{1}$$

where ω_{ce} is the electron gyrofrequency, ω and $k = |\vec{k}|$ are, respectively, the whistler-mode wave frequency and wave number (it is assumed that $\vec{k} \| \vec{B}$), and $v_{\rm step}$ is the absolute value of the step velocity which is changing along the generation region due to the magnetic-field inhomogeneity and interaction with waves.

[13] The strict nonlinear theory of the BWO generation mechanism in an inhomogeneous magnetic field is rather complicated and has been analyzed only in the case of a narrow dynamic frequency spectrum [*Demekhov and Trakhtengerts*, 2005]. That analysis demonstrates a spike-like generation regime, which can explain many features of chorus generation [*Trakhtengerts et al.*, 2004].

[14] We suggest here that the formation of the chorus spectrum is due to the continuous erosion of the step feature during the generation of a single chorus element. As we discuss below, the variation in the step velocity inside the generation region can be described by the equation

$$\frac{\mathrm{d}v_{\rm step}}{\mathrm{d}t} \approx \frac{\mathrm{d}^2 z_g}{\mathrm{d}t^2} \approx -\frac{\Omega_{\rm tr}^2}{2\pi k} - \frac{v_{\perp 0}^2}{2B v_{\rm step}} \frac{\mathrm{d}B}{\mathrm{d}z} \tag{2}$$

(hereafter, the signs in equations correspond to the case of electrons moving in the $+z$ direction and waves propagating in the $-z$ direction). The second term on the right-hand side of equation (2) describes the magnetic mirror force, and the first nonlinear term is proportional to the square of the trapping frequency of an electron near cyclotron resonance in the potential well of the wave field [*Karpman et al.*, 1974]:

$$\Omega_{\rm tr} = (k v_{\perp 0} \omega_{B\sim})^{1/2}, \qquad \omega_{B\sim} = eB_{\sim}/mc, \tag{3}$$

where $B_{\sim}$ is the wave magnetic field and $v_{\perp 0}$ is the characteristic velocity component perpendicular to $\vec{B}$. This term describes the step distortion (erosion) due to the development of the sideband instability [*Denavit and Sudan*, 1975]. The physical sense of this term is as follows. Generation of waves leads to the bunching of resonant electrons and spreading of the step edge over the interval of parallel velocities $\Delta v_{\rm step} \sim \Omega_{\rm tr}/k$, which corresponds to the width of the wave potential well. This process occurs during about the half-period of oscillations of the trapped electrons in the wave field, $\Delta t \sim \pi/\Omega_{\rm tr}$. The combination $\Delta v_{\rm step}/\Delta t$ yields exactly the first term on the right-hand side of (2). Here, we consider a simple model in which $B_{\sim}$ is fixed and can be obtained from experimental data. Adoption of such a nonlinear term in equation (2) yields an estimate of the frequency drift consistent with both case-studies of Cluster data [*Trakhtengerts et al.*, 2004] and statistical studies of Magion 5 data [*Titova et al.*, 2003]. Moreover, the existence of such a distortion was demonstrated in numerical calculations by [*Nunn*, 1990] for triggered emissions, though its temporal evolution was not studied.

[15] Equations (1)–(3) yield the spatial coordinate z_g and time t_g of generation of every frequency $f = \omega/(2\pi)$ in the dynamic spectrum of one chorus element. The values of z_g and t_g are related by

$$t_g = t_0 + \int_{-l/2}^{z_g} \frac{d\xi}{v_{\rm step}(\xi)} \tag{4}$$

where $t = t_0$ corresponds to the moment when the electrons enter the generation region ($z_g = -l/2$). At an arbitrary point

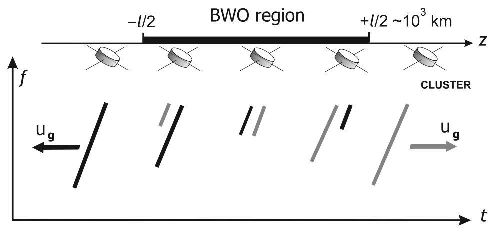

Figure 3. Diagram showing the formation of chorus elements. Black and gray slanted lines on the sketch of the dynamic spectrogram show the elements propagating to the left and right, respectively. Tiny spacecraft icons above the elements show positions along the field line where they should be observed. The horizontal black line at the top denotes the chorus source region.

z along the wave propagation path, the dynamic spectrum of a chorus element can be found from the relation

$$t(\omega, z) = t_g + \int_z^{z_g} \frac{d\xi}{v_{\rm gr}(\omega, \xi)} \qquad (5)$$

where $v_{\rm gr}$ is the wave group velocity (note that the element is seen only for $t > t_g$, which implies $z_g > z$ if the waves propagate in the $-z$ direction).

[16] A schematic picture for the formation of the dynamic spectrum of a particular chorus element according to relations (1)–(5) is shown in Figure 3. Here, we consider the case in which the nonlinear term in equation (2) dominates. This is typical for the Cluster data obtained in the source region where we observe large chorus amplitudes. In this case, according to (1)–(2), the minimum frequency f_0 is generated at the entrance of electrons to the BWO region (at $t = t_0$, about 1000 km from the magnetic equator), where the step velocity is maximum. The scheme in Figure 3 clearly shows that complete chorus elements can only be seen outside and near the boundaries of the source region, while near its center, only the upper-frequency parts of the elements are seen. This is in agreement with the reported observations.

[17] For a quantitative illustration, we use the parameters of the source region and chorus emissions corresponding to the event of 18 April 2002, i.e., the source size $\Delta z \approx 2000$ km, chorus amplitudes $B_\sim \approx 100$ pT, and the cold plasma density $N_c \simeq 2$ cm^{-1} at $L \simeq 4.4$ [*Santolik et al.*, 2003]. This yields rather high resonant parallel energy $W_\parallel = mv^2_{\rm step}/ \simeq 62$ keV. Assuming $v_{\perp 0}/v_{\rm step} \sim 1$, we find that the nonlinear term in equation (2) exceeds the mirror-force term two times even at the source boundary, while this domination becomes much stronger closer to the center. Neglecting the mirror force, one easily obtains from equation (2):

$$v^2_{\rm step} \simeq v^2_{\rm step\,0} - \pi^{-1} v_{\perp 0}(z - z_0)(B_\sim/B), \qquad (6)$$

where the subscript "0" refers to the source boundary at the electrons-entrance side. This clearly demonstrates a decrease in the step velocity and the corresponding increase in the generated frequency as the electrons move in the source ($z > z_0$). Combining (6) with equations (1), (4), and (5), it is easy to obtain an estimate for the frequency drift, which is about 10^4 Hz/s for the parameters used above and is consistent with the corresponding Cluster data.

4. Conclusions

[18] A large volume of Cluster data on night-time chorus emissions (3000 discrete elements), obtained on two geomagnetically active days of 18 April 2002 and 31 March 2001 in and near the chorus source region, was used to investigate the dependence of a frequency range occupied by an individual chorus element on the position of the measurement point. We have revealed a lack of lower frequencies in the central part of the generation region. This effect can be explained on the basis of the backward-wave oscillator model of chorus generation, in which the chorus is generated by electrons with a step-like deformation on the distribution function in parallel velocities, and the chorus frequency is related to the instantaneous parallel velocity at the step. The lack of lower frequencies is due to the step erosion in the process of electron interaction with chorus waves. Due to this erosion, there is a difference between chorus elements observed in the middle of the source and close to its boundaries. For the typical case of a rising-tone chorus element, its lower-frequency part is generated downstream with respect to the wave propagation and, hence, is undetectable by a receiver located upstream within the source region. Taking into account the wave packets propagating in both directions, this results in the lack of lower frequencies in chorus spectra obtained in the central part of the source. Quantitative estimates based on the measured chorus amplitudes yield reasonable values of the frequency drift in a chorus element, which provides additional support to the proposed scheme.

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