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CONNECTION BETWEEN AMBIENT DENSITY FLUCTUATIONS AND CLUMPY LANGMUIR WAVES IN TYPE III RADIO SOURCES

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ABSTRACT

A recent stochastic-growth theory of clumpy Langmuir waves in type III sources is shown to imply that the clumps will have the same size distribution as the ambient low-frequency density fluctuations in the solar wind. Spectral analysis of Langmuir-wave time series from the *ISEE 3* plasma wave instrument confirms this prediction to within the uncertainties in the spectra. The smallest Langmuir clump size is inferred to be in the range 0.4–30 km in general, and 2–30 km for beam-resonant waves, and it is concluded that the diffusion of waves in the source is anomalous.

Subject headings: plasmas — solar wind — Sun: radio radiation

1. INTRODUCTION

A major theoretical difficulty with the generation of Langmuir waves by type III electron beams was raised by Sturrock (1964), who argued that beams should propagate only a few kilometers before losing virtually all their energy to the waves, whereas they are actually observed to travel distances of several AU. Subsequent simulations of quasi-linear relaxation of the beam showed that energy transfered to waves at the head of the beam is reabsorbed by electrons further back, reducing the net energy loss and allowing the beam to propagate to the observed distances (Grognard 1985; Melrose 1990; Muschietti 1990). These simulations also reproduced the approximate form of the electron distributions observed at 1 AU (Grognard 1985).

Two shortcomings exist in the simple quasi-linear-relaxation model outlined above. First, observations have shown that ambient low-frequency density fluctuations in the solar wind exist at a level sufficient to scatter Langmuir waves out of resonance with the beam faster than they can grow on average (Celnikier et al. 1983, 1987; Muschietti et al. 1985). Second, the model does not explain the extreme clumpiness of the observed Langmuir waves, which show structure down to the shortest scales resolved (Gurnett & Anderson 1976; 1977; Gurnett et al. 1980; Lin et al. 1981, 1986). Several solutions to these problems have been proposed: (1) Nonlinear instabilities leading to wave collapse have been suggested as the clumping mechanism. However, even fields well below the collapse threshold are extremely clumpy, and the relative peak levels of ion-sound and Langmuir waves are actually consistent with ion-sound generation via three-wave decay (Muschietti 1990). (2) Density fluctuations may be reduced during type III events, thereby permitting growth. However, such fluctuations appear to be ubiquitous, and no mechanism has been suggested to suppress them during type III events. (3) Muschietti et al. (1985) and Melrose et al. (1986) noted that scattering is ineffective for fluctuations having the form of narrow density channels aligned along the beam direction. However, more recent observations appear to rule out any such extreme anisotropy (Celnikier et al. 1987). (4) Melrose & Goldman (1987) postulated that very large growth rates occur in localized regions where the density fluctuations have a reduced amplitude. The required suppression is very large, but this scenario has the advantage that it preserves the successes of quasi-linear theory: on average the evolution is the same whether or not the waves are clumped (Melrose & Cramer 1989). (5) Smith & Sime (1979) showed that density fluctuations favor growth along some rays, but not others, thereby causing clumping. (6) Incorporating several of the above ideas in a new model, Robinson (1992) argued that the beam propagates in an average state close to marginal stability, but that both the beam and the waves are perturbed by ambient density fluctuations. This leads to stochastic growth in which the growth rate fluctuates and the logarithm of the wave energy density undergoes a random walk, with positive growth only in localized regions. He showed that this can account for wave growth under type III conditions and, qualitatively, for the wave clumping and distribution of field strengths (Robinson et al. 1987).

In this *Letter* we show that the stochastic-growth model of type III sources predicts Langmuir clumps to have the same size distribution as the ambient density fluctuations. We use time series of Langmuir-wave field strengths from the joint TRW-JPL-Iowa plasma wave instrument on *ISEE 3* (Scarf et al. 1978) to test this prediction by calculating the wavenumber power spectrum of the clumps. This spectrum is compared with the density-fluctuation spectrum (Celnikier et al. 1983, 1987), and it is found that they are indeed consistent to within their uncertainties. This result implies that the effects of density fluctuations must be incorporated in type III theories and supports the stochastic-growth theory in particular. It also enables us to constrain the inner and outer scales of the clump-size distribution and to infer that wave diffusion will be anomalous.

2. THEORY

Quasi-linear-relaxation calculations imply that type III beams evolve toward a state of marginal stability in which the net growth of Langmuir waves just balances net losses (Grognard 1985). In Robinson's (1992) model ambient density fluctuations perturb this state by allowing growth only in localized "growth regions" where the density gradient is approximately parallel to the beam direction, and, hence, refraction of Langmuir waves out of resonance is suppressed. (This purely geometric condition does not require the level of density fluctuations to be reduced.) Both beam and waves develop inhomogeneities on scales comparable with those of the density fluctuations, leading to local fluctuations in the growth rate as the beam passes by (Robinson 1992). The logarithmic energy density in the waves is thus predicted to undergo a random walk, with net growth while the waves remain in a growth region and net damping otherwise.

A distribution of fluctuation scales must be incorporated when investigating the spatial structure of the Langmuir clumps. We begin by considering a single approximately isotropic clump of characteristic amplitude n (the deviation from the mean density), size ℓ, and gradient n/ℓ, and determine how the growth of waves scales with these quantities. Robinson (1992) showed that the stochastic interaction of an inhomogeneous beam with an ensemble of such clumps can be represented by an effective growth rate whose mean value (averaged over all space) is zero when marginal stability is attained. The interaction of the inhomogeneous beam with waves in a single clump can be represented in the same way, but with a nonzero characteristic growth rate

$$\Gamma \propto |\nabla n - (\nabla n)_0| , \tag{1}$$

where $(\nabla n)_0$ is the gradient corresponding to local marginally stable balance between growth and refraction out of resonance. (The latter process gives an effective instantaneous damping due to refraction of order $k^{-1} |\partial \boldsymbol{k}/\partial t| \propto |(\nabla n)_0|$, where $\boldsymbol{k}$ is the wave vector (Melrose et al. 1986).] The growth rate (1) depends on density gradients and thus scales as n/ℓ. Similarly, the time taken for waves of group velocity v_g to cross the clump is ℓ/v_g. Hence, the total number G of e-foldings experienced by the growing waves scales as

$$G \propto \Gamma \ell / v_g \propto n/v_g . \tag{2}$$

Equation (2) implies that the size distributions of density clumps and clumps of G are proportional; that is, $P_n(\ell) \propto P_G(\ell)$. When observed in situ, these distributions correspond to distributions of clump durations $t \approx \ell/v_g$ as the solar wind sweeps by the receiver. Suppose a spacecraft encounters an ensemble of clumps with the idealized distribution

$$P_G(t) \propto P_n(t) \propto t^{-a} , \tag{3}$$

for $t_{\min} < t < t_{\max}$. The observed time history of G (or, equivalently, $\log E$) is the sum of contributions from individual clumps of duration t. If we assume these clumps are uncorrelated, the total power spectrum $S_G(\omega)$ is the integral [weighted by $P_G(t)$] over the spectra $S_G(\omega; t)$ of the individual clumps of characteristic duration t. This gives

$$S_G(\omega) \propto \int_{t_{\min}}^{t_{\max}} d\tau\, \tau^{-a} S_G(\omega; \tau) . \tag{4}$$

For $\omega \ll 1/\tau$ one has $S_G(\omega; \tau) \approx |\int_{-\infty}^{\infty} dt f(t, \tau)|^2 \propto \tau^2$, where $f(t, \tau)$ is the time profile of the corresponding clump of characteristic duration τ and unit characteristic height. Hence, for $1/t_{\max} \ll \omega \ll 1/t_{\min}$ and $a < 3$,

$$S_G(\omega) \propto \int_{t_{\min}}^{1/\omega} d\tau\, \tau^{2-a} , \tag{5}$$

$$\propto \omega^{a-3} , \tag{6}$$

a result that is independent of the shape of the clumps. The spectrum plateaus for $\omega \lesssim 1/t_{\max}$ and depends on the shape of the individual clumps for $\omega \gtrsim 1/t_{\min}$.

Two effects must be considered when comparing the spectrum (6) with observations: (1) The effect of the instrumental sensitivity limit is to flatten the spectrum, a trend that becomes more pronounced as this limit moves to higher values of $\log E$. (2) Observed spectra (see § 3) often have a double power-law form, with

$$S_G(\omega) \propto \begin{cases} \omega^{a-3} , & \omega \ll \omega_0 , \\ \omega^{b-3} , & \omega_0 \ll \omega , \end{cases} \tag{7}$$

where ω_0 is the breakpoint. This corresponds to a size distribution

$$P_G(t) \propto \begin{cases} t^{-a} , & t_0 \ll t , \\ t^{-b} , & t \ll t_0 , \end{cases} \tag{8}$$

with $\omega_0 t_0 \approx 1$. For $a \neq b$ the spectrum does not have a power-law form near ω_0, an effect that must be accounted for, particularly when comparing with observations that cover only a limited range of frequency around ω_0. The procedure we use to relate sensitivity-limited double power-law spectra to the underlying spectra and clump-size distributions is to construct a Monte Carlo distribution of clumps numerically using equation (8) with a sharp breakpoint at t_0, impose the appropriate sensitivity limit, and calculate the Fourier transform. The parameters of the Monte Carlo distribution are then varied to optimize the fit to the observed spectrum up to the Nyquist frequency.

3. COMPARISON OF THEORY AND OBSERVATIONS

The joint TRW-JPL-Iowa plasma wave instrument on *ISEE 3* returns field measurements near the plasma frequency at 0.5 s intervals. These time series show spiky Langmuir waves during type III events, with structure down to the shortest times resolved (see Lin et al. 1981, 1986 for figures). Here we analyze the time series from three bursts observed at 1 AU that have previously been extensively studied by other means, namely the events of 1979 February 17 (Lin et al. 1981), 1979 February 8, and 1979 March 11 (Lin et al. 1986).

Figure 1 shows the Fourier transform of the time series of $\log E$ from the 31.6 kHz channel during the 1979 February 17 event, between 1930 UT and 2200 UT (solid curve; the dashed curve is discussed below). The sensitivity to Langmuir waves was limited by the simultaneous detection of the electromagnetic emission that constitutes the type III radio burst. Hence, fields below the resulting sensitivity limit $10^{-5.3}$ V m^{-1} were reassigned this value prior to Fourier transforming. The resulting spectrum has a double power-law form, but the exponents must be corrected for the effects of the sensitivity limit and proximity to ω_0 before a and b can be determined from them.

Figure 2 shows the underlying spectrum corresponding to that in Figure 1. It was calculated by numerically fitting the curve in Figure 1 with the spectrum (dashed curve in Fig. 1) of a Monte Carlo distribution of clumps of the form (8) using the appropriate sensitivity limit, then removing this limit to obtain the underlying spectrum. The resulting spectrum is of the form (7), with $3 - a = 1.55 \pm 0.1$, $3 - b = 0.0 \pm 0.1$, and $\omega_0 = 0.8 \pm 0.2$ s^{-1}. The first five lines of Table 1 list $3 - a$, $3 - b$, and ω_0 for the events under consideration, while the last two lines give, respectively, the overall parameter ranges observed

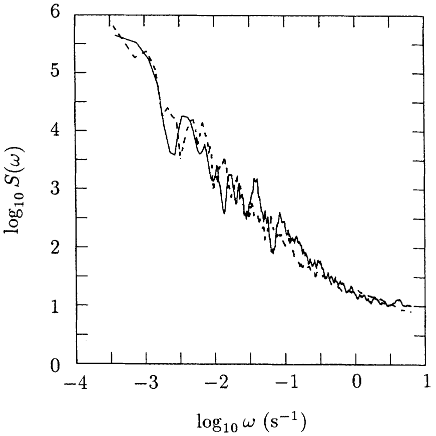

FIG. 1.—Frequency spectrum (*solid curve*, arbitrary units) from the 31.6 kHz channel for the 1979 February 17 event, smoothed over a range 5% either side of the nominal frequency for plotting. The dashed curve shows the corresponding Monte Carlo spectrum.

(allowing for uncertainties) and the corresponding ranges for density spectra (Celnikier et al. 1987).

Our spectral analysis leads to the following results:

1. Table 1 shows that the parameters obtained from different channels in the February 8 and February 17 events are consistent with one another. The density and log-E spectra are also mutually consistent, given the individual uncertainties and event-to-event variation. Most importantly, the breakpoints, which depend least on the details of the model, are in agreement, with $\ell_0 = 2\pi v_{sw}/\omega_0 = 2500$–$6000$ km for a solar wind velocity of $v_{sw} = 400$ km s^{-1}. The log-E spectra are slightly flatter than the density spectra, but this is not statistically significant, given the small number of events and the spread of values in each data set. Finally, the typical spectral parameters are consistent with those obtained using interplanetary-

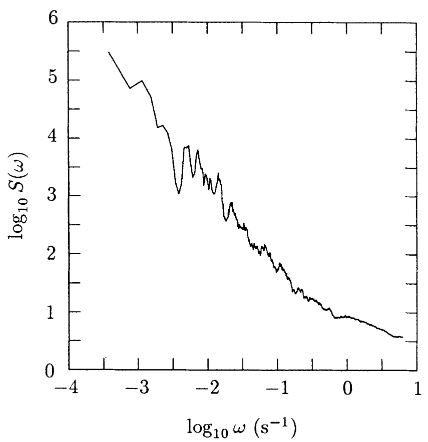

FIG. 2.—Monte Carlo spectrum from Fig. 1 (arbitrary units) corrected for instrumental effects (*see text*) and smoothed as in Fig. 1.

TABLE 1

SPECTRAL PARAMETERS FOR TYPE III EVENTS AND AMBIENT DENSITY FLUCTUATIONS

Event	Channel (kHz)	$3 - a$	$3 - b$	ω_0 (s^{-1})
Feb 8	17.8	1.3 ± 0.1	0.75 ± 0.1	0.4 ± 0.1
Feb 8	31.6	1.45 ± 0.1	0.8 ± 0.1	0.4 ± 0.1
Feb 17	17.8	1.45 ± 0.1	0.0 ± 0.1	0.8 ± 0.2
Feb 17	31.6	1.55 ± 0.1	0.0 ± 0.1	0.8 ± 0.2
Mar 11	17.8	1.3 ± 0.1	0.5 ± 0.1	0.5 ± 0.2
Range of values		1.2–1.65	−0.1 to 0.9	0.3–1.0
Density spectrum		1.5–1.8	0.3–0.95	0.4–1.0

scintillation measurements, for example (see the discussion by Celnikier et al. 1983).

2. Result (1) implies that ambient low-frequency density fluctuations *are* present during type III events and have similar properties to those seen at other time periods (Celnikier et al. 1983, 1987). Hence, theories of type III events must account for the resulting effective dissipation by scattering (e.g., Robinson 1992).

3. No plateauing is observed at small ω, implying that larger scales are present than $\ell = 2\pi v_{sw}/\omega_{min} \approx 6 \times 10^6$ km, consistent with the results of Celnikier et al. (1983), which extended to scales of $\sim 10^6$ km, and with interplanetary scintillation data to $\sim 4 \times 10^6$ km (Celnikier et al. 1983). Neither is any shape change observed at the largest ω, implying that the smallest structures are not resolved (i.e., $\ell_{min} < 400$ km), in accord with Celnikier et al. (1987) who inferred a minimum scale $\ell_{min} < 30$ km. Strong dissipation of short-wavelength Langmuir waves restricts the minimum full width of a clump to values $\gtrsim 40\lambda_D \approx 400$ m. We conclude

$$0.4 \text{ km} < \ell_{min} < 30 \text{ km} , \tag{9a}$$

in general, and

$$2 \text{ km} < \ell_{min} < 30 \text{ km} , \tag{9b}$$

for beam-resonant waves, for which the wavelength $2\pi v_b \lambda_D/V$ is the appropriate lower bound (here we assume $\lambda_D = 10$ m, $v_b = 3 \times 10^7$ m s^{-1}, and $V = 10^6$ m s^{-1}). Observations with less than 1 ms and less than 10 ms resolution would be needed to resolve structures at the lower bounds in equations (9a) and (9b), respectively.

4. The large-ℓ behavior $P_G(\ell) \sim \ell^{-a}$ leads to formal divergences in the averages $\langle \ell \rangle$ and $\langle \ell^2 \rangle$ for $a < 2$ and $a < 3$, respectively. Physically, these divergences are prevented by the existence of an outer scale ℓ_{max}, but lead to anomalous diffusion for the observed values of $a \approx 1.5$ (Scher et al. 1991; Scher & Montroll 1975).

For Langmuir waves the outer scale arises because the waves can only grow in resonance with the beam for a distance $\ell_{max} \approx r\,\Delta\omega/\omega_p \approx 3rV^2\,\Delta v_b/(2v_b^3)$, where r, $\Delta\omega$, v_b, Δv_b, and V are the distance from the Sun, emission bandwidth, beam velocity, beam velocity spread, and thermal velocity, respectively (Melrose et al. 1986); beyond this point the waves are damped. This gives $\ell_{max} = 2 \times 10^3$–$4 \times 10^4$ km for typical parameters at $r = 1$ AU. For $1 < a < 2$ and $2 < b < 3$, we find

$$\langle \ell \rangle \approx \frac{b-1}{b-2}\,\ell_{min} , \tag{10a}$$

$$\langle \ell^2 \rangle \approx \frac{b-1}{3-a}\,\ell_{min}^{b-1}\,\ell_0^{a-b}\,\ell_{max}^{3-a} , \tag{10b}$$

where some small terms have been neglected. Equations (10a) and (10b) give $\langle \ell \rangle = 6$–100 km, and $\langle \ell^2 \rangle = (30$–$700)^2$ km^2 for $a = 1.5$, $b = 2.5$, $\ell_0 = 4000$ km, $\ell_{\min} = 2$–30 km, and $\ell_{\max} = 10^4$ km. Note that clumps larger than $\ell_{\max}$ permit simultaneous growth in more than one adjacent band of width $\Delta\omega$. The clump is then observed as a whole because of the relatively large bandwidth $\Delta\omega_r$ of the receiver. The maximum clump size that can be observed in this way by *ISEE 3* is $\sim r\,\Delta\omega_r/\omega_p \approx 2 \times 10^7$ km at 1 AU.

Anomalous diffusion of Langmuir waves is significant because it implies that a larger fraction of paths have $\ell \gg \langle \ell \rangle$ than for a Gaussian distribution of length scales, and stochastic growth is thus more effective (the paths on which growth does occur dominate the wave energy density, so any increase in the number of long paths is significant). In contrast, those waves that do scatter are strongly damped. Scattering and diffusion of electromagnetic waves are also expected to be anomalous.

5. Celnikier et al. (1987) found only two sets of data from several years of *ISEE 1*/*ISEE 2* observations that could be used to determine density spectra. Spectra of log E provide a possible probe of density fluctuations (although not of the overall normalization of the spectrum) for which more data sets are available as input. The *ISEE 3* data do not extend to as high a frequency as Celnikier et al.'s (1987) data, and they are, of course, restricted to the periods of type III events, but this method only requires observations by one spacecraft and could be applied to data other than from *ISEE 3*.

6. The results obtained here differ from those of a stochastic-growth theory based on density-fluctuation suppression in localized regions, of size ℓ (implying $\Gamma \sim n$, rather than $\Gamma \sim n/\ell$), which gives $G \sim n\ell/v_g$ in place of equation (2), and, hence, $P_G(t) \sim tP_n(t)$. Such a theory predicts the spectral exponents of $S_G(\omega)$ to be larger by unity than those of $S_n(\omega)$, contrary to observation. This result implies that it is alignment of ∇n with the beam direction, rather than absence or suppression of fluctuations, that is the principal determinant of growth regions.

4. CONCLUSION

We have found that the spectra of log E during three type III events are consistent with ambient density spectra measured at other times to within their uncertainties. Our generalization of the stochastic-growth theory of Robinson (1992) to incorporate these fluctuations correctly predicts correlations of the form observed, when growth regions are determined by alignment of ∇n with the beam direction to suppress refraction. These results imply that ambient solar-wind density fluctuations persist during type III events, influencing the clumping of Langmuir waves and consequent nonuniformities in the electron beam. The observed density spectrum enables the minimum Langmuir clump size to be constrained to the range 0.4–30 km in general, and 2–30 km for beam-resonant waves, and implies that the diffusion of waves is anomalous, thereby enhancing the effectiveness of stochastic growth.

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