

Modeling of Whistler Ray Paths in the Magnetosphere of Neptune

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A three-dimensional ray tracing program is used to investigate the propagation and dispersion of lightning-generated whistlers in the magnetosphere of Neptune. A simple empirical magnetospheric plasma model based on observational constraints is used for the plasma density distribution, and an offset tilted dipole model is used for the magnetic field. Source positions for the whistlers are chosen along magnetic field lines with $L < 3$ in agreement with observations. Whistlers with frequencies of about 10 kHz generally propagate very near the resonance cone angle for most of the ray path, which produces dispersions considerably larger than those predicted for quasiparallel propagation. Dispersions computed using our "standard" plasma density model are over 30,000 s $\sqrt{\text{Hz}}$ for "one-hop" whistlers which is comparable to observed values. In general, the whistlers were found to propagate about 10° in magnetic longitude during one hop and to cross L shells. These results strongly support the existence of whistlers at Neptune. The large dispersions are attributed to the large cold plasma population at Neptune which leads to propagation very near the resonance cone angle for a large part of the ray path.

INTRODUCTION

During the Voyager 2 flyby of Neptune, which occurred on August, 25, 1989, the plasma wave instrument detected 16 "whistlerlike" events near the magnetic equator at radial distances ranging from 1.30 to 1.99 R_N [Gurnett *et al.*, 1989, 1990]. The frequency ranged from 6.1 to 12.0 kHz, and the dispersions fit the Eckersley law for lightning-generated whistlers. The existence of whistler mode emissions (which propagate at frequencies less than the minimum of either the gyrofrequency, f_c , or the plasma frequency, f_p) sets a limit on the density at the point of observation. The magnetic field was directly measured at the point of observation, and the local gyrofrequency during the observations was at least 100 kHz. Belcher *et al.* [1989] have reported that plasma density measurements near closest approach (CA) are typically 0.01 to 0.1 cm^{-3} with a maximum of 1.4 cm^{-3} near the magnetic equator. These values are dependent on ion composition which is uncertain. However, the instrument was probably not sensitive to cold plasma because the Faraday cups were not favorably oriented at the time of CA (R. McNutt, private communication, 1990). We also note that Barbosa *et al.* [1990] have estimated the density at this time based on plasma wave measurements of ion cyclotron harmonics to be in the range of 1 to 10 cm^{-3} , again dependent on the ion composition. Sawyer *et al.* [1990] reported what appear to be electron cyclotron harmonics at this time in the Planetary Radio Astronomy data, which may also indicate higher densities based on the cyclotron-maser instability. The topic of plasma density was discussed at great length by Gurnett *et al.* [1990], and led the authors to suggest that the signals were in fact whistlers. Gurnett *et al.*

believe the density at the point of observation must be at least 30 to 100 cm^{-3} .

One result that is difficult to understand is the extremely large dispersions, typically 26,000 s $\sqrt{\text{Hz}}$. The dispersion is defined by the Eckersley law, $t = D/\sqrt{f}$, where t is the travel time, f is the frequency and D is a constant called the dispersion. For low frequencies and propagation parallel to the magnetic field the dispersion constant D is given by

$$D = (1/2c) \int \frac{f_p ds}{\sqrt{f_c}} \quad (1)$$

where ds is elemental distance along the ray path. Based on minimum and maximum estimates of the density in the magnetosphere, Gurnett *et al.* calculated that path lengths ranging from ~ 50 to over 2000 R_N are required to achieve the observed dispersion. While whistlers observed at Earth are in rare cases known to have bounced as many as 200 times, it is difficult to imagine an electromagnetic wave surviving such long path lengths without suffering significant damping, scattering, or other processes that would strongly degrade the amplitude.

In this paper we present a preliminary study of whistler propagation based on three-dimensional ray tracing. We emphasize that these are initial results and more work needs to be done to understand the details of the observations. The results show that very large dispersions are obtained even without multiple bounces. The large dispersions are the result of propagation very near the resonance cone angle for a large part of the ray path. Such dispersions are considerably larger than predicted by the Eckersley law, which is not valid at large wave normal angles.

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PLASMA AND MAGNETIC FIELD MODELS

The magnetic field model is an offset tilted dipole which is a refinement (OTD2) of an earlier model developed by Ness *et al.*

[1989]. This model is based on a dipole, with magnetic moment of $\mu = 0.13 \text{ GR}_N^3$ tilted by about 45° with respect to the rotation axis and is offset from the center by $0.55 R_N$. This model is known to be quite accurate along the spacecraft trajectory at distances greater than about $3 R_N$, but may have significant deviations from the real field close to the planet (N.F. Ness, private communication, 1990). The large offset produces a strong asymmetry in the field strength between northern and southern magnetic hemispheres, and the tilt produces a strong longitudinal asymmetry. As will be shown, these asymmetries have noticeable effects on whistler propagation.

The plasma model consists of an ionospheric model and a magnetospheric model. The ionospheric model is due to *Shinagawa and Waite* [1989]. Specifically we have used the model represented by curve "a" in their Figure 1, page 946. This was their "standard case" model and produced peak densities of about $6 \times 10^4 \text{ cm}^{-3}$ at an altitude of about 1700 km. This model differs from the results of the Voyager radio science observations [Tyler *et al.*, 1989]. The maximum density obtained from the radio occultations was about $2.5 \times 10^3 \text{ cm}^{-3}$ at an altitude of about 1400 km, but it is to be noted that these observations were made near the terminator. Most likely the electron densities are much higher on the dayside of Neptune, particularly near the subsolar point. We used a simple linear fit to the logarithmic function of Shinagawa and Waite as follows:

$$n_{\text{ionos}} = 10^{m/r} n_i \quad (\text{cm}^{-3}) \quad (2)$$

with $m = -3.16 \times 10^4$, $n_i = 13.18$, and r the radial distance in kilometers. For the magnetospheric electron density we have introduced a simple empirical model with a radial dependence only:

$$n = n_0 + C e^{-k/(R_N - r)^p} \quad (3)$$

with $C = 99.91$, $k = 0.567$, $n_0 = 0.09$, and $R_N = 24,765 \text{ km}$. The constants were selected to produce a density of about 30 cm^{-3} at $L = 3$. This value is approximately midrange between the estimate of the density range suggested by *Gurnett et al.* [1990]. For this initial study no local time dependence was incorporated

in the density model. We have also examined a modification of this magnetospheric model by introducing a latitudinal dependence with a peak density at the geographic equator. The specific form of the latitudinal dependence is given by

$$n' = n[1 + A \cos(\lambda)] \quad (4)$$

where n' is the density, A is a variable, and λ is the geographic latitude. A plot of the density versus distance along a sample magnetic field line for both equations (3) and (4) is shown in Figure 1.

RAY TRACING

Ray tracing calculations were performed using the three-dimensional code that has been discussed in the past [cf. *Menietti et al.*, 1990]. This code is based on the cold plasma theory of *Stix* [1962] and *Haselgrove's* [1955] set of first-order differential equations. Early in the study it was discovered, however, that whistler wave propagation near the resonance cone was occurring because of the high ionospheric and magnetospheric densities. When this happens, the index of refraction becomes so large that numerical instabilities prohibit further integration. To circumvent this problem, a modification of the basic equations was introduced on the basis of the scheme of *Cerisier* [1970], also utilized by *Cairo and Lefeuvre* [1986]. The fundamental change was the introduction of the following logarithmic derivative:

$$\frac{d}{ds} \ln(\cos^2 \Psi - \cos^2 \psi_r) = -2 \sin \psi_r \cos \psi_r \frac{d\psi_r}{ds} - \frac{d \cos^2 \psi_r}{ds} / (\cos^2 \psi_r - \cos^2 \Psi) \quad (5)$$

with

$$\frac{d \cos^2 \psi_r}{ds} = \frac{1}{(S - P)} \frac{dS}{ds} - \frac{S}{(S - P)^2} \left[\frac{dS}{ds} - \frac{dP}{ds} \right]$$

where Ψ is the wave normal angle, s is the ray path, and ψ_r is the resonance cone angle, which according to *Stix* [1962] is given by

$$\tan^2(\psi_r) = -P/S \quad (6)$$

with P and S defined by *Stix* [1962]. As discussed in detail by *Cerisier* [1970], because of the slow logarithmic dependence, equation (5) allows the evaluation of Ψ and numerical integration near the resonance cone angle.

The Eckersley law dispersion given by equation (1) is only valid for low frequencies and small wave normal angles, such that $ff_c \ll \cos(\psi)$. This assumption is called the quasi-parallel approximation [*Helliwell*, 1965]. The quasi-parallel approximation fails for propagation near the resonance cone angle, particularly when the ray path is at larger distances ($r \sim 2 R_N$) from Neptune. As we describe below, we initially calculate the time delay, T , for a large number of rays at $f = 10 \text{ kHz}$ in order to obtain candidate source positions. To compute T we use the formula $T = \int ds/v_g$ where v_g is the group velocity. We later estimate D by comparing T for a number of ray paths, from the same source point, at different frequencies. The initial wave normal angles are adjusted such that the ray paths for each frequency intersect one point in the magnetosphere.

RESULTS

The whistlers observed by Voyager 2 all occurred within about 35° of the magnetic equator and at $1.4 < L < 2$. The L values were obtained from N.F. Ness (personal communication, 1991).

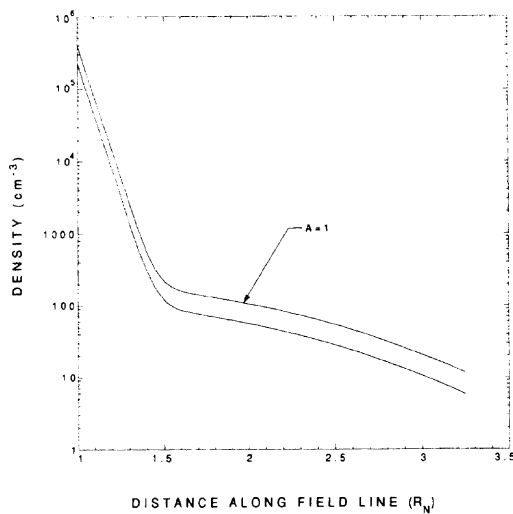


Fig. 1. Density versus distance along a magnetic field line. The footprint of the magnetic field line chosen was $\lambda = 40^\circ$, $A = 140^\circ$. The curves are for equation (4) with $A = 0$ and $A = 1$.

Using the OTD2 model we identified source positions near the surface of Neptune for which $L < 3$. Generally speaking there were two such positions along each longitude meridian of Neptune, one in each hemisphere. Initially we confined our attention to source positions located nearer the north (weaker) magnetic pole because the lower hybrid frequency, f_{LH} , was much lower in this case. This is true because

$$1/f_{LH}^2 = 1/f_{ci}^2 + 1/f_{pi}^2 + 1/(f_{ci} f_c) \quad (7)$$

and is strongly dependent on the local field strength through f_c . In this equation, f_{ci} and f_{pi} are the ion cyclotron frequency and ion plasma frequency, respectively. As an example, for a source near the north magnetic pole ($\lambda = 5^\circ$, $\Lambda = 140^\circ$), $f_{ci} \sim 8.5 \times 10^3$ kHz, $f_{pi} \sim 54$ kHz, $f_c \sim 132$ kHz, and $f_{LH} \sim 3.1$ kHz. In contrast for a source near the much stronger south magnetic pole ($\lambda = -30^\circ$, $\Lambda = 250^\circ$), $f_{ci} \sim 3.1 \times 10^2$ kHz, $f_{pi} \sim 54$ kHz, $f_c \sim 1710$ kHz, and $f_{LH} \sim 32.1$ kHz. When $f < f_{LH}$, the index-of-refraction surface is closed, and reflection occurs when f approaches f_{LH} or if the wave normal angle approaches 90° . A wave propagating away from the surface in a region where $f_{LH} \gg f$ (where magnetic field strength is large) can be trapped in the ionosphere, and a wave from the magnetosphere can be reflected back. For $f > f_{LH}$, the index-of-refraction surface changes to an open surface with a discontinuity at the resonance cone angle.

The rays were launched at an altitude of 1700 km, near the peak ionospheric density, at a wave normal angle that corresponded to a wave vector directed radially outward (vertical) from the planet. A vertical initial wave normal direction is used because whistler mode waves from lightning are expected to be refracted to near vertical due to the large index of refraction in the ionosphere. Generally, the initial wave normal angle was about 25° to 30° . The frequency was 10 kHz, and source positions were chosen every 10° of longitude. Source positions with the smallest local magnetic field strength usually produced the largest time delays. This follows approximately from the equation for D which shows that the dispersion is inversely proportional to $\sqrt{f_c}$ (see equation (1)). In Table 1 we list a summary of results for various source positions and for two density models as identified in the last column. The angle β is the azimuthal angle about the magnetic field; i.e., this angle determined the plane in which the ray was launched. The approximate L shell for a particular ray path as it crossed the magnetic equator is designated as L_{avg} . This L value is typically different from the L value of the source point because of deviations of the ray path from a fixed L shell.

The time delays listed in the table are the total integrated values at a point near the magnetic equator before the ray propagates to the opposite hemisphere. Many of the travel times in Table 1 are much larger than comparable values calculated at Earth or at Jupiter. The reason for the large travel times is twofold. First, the enormous dimensions of Neptune produce a ray path much longer than any obtained at Earth, which causes a corresponding increase in the dispersion. Second, the wave normal angles of the rays are very near the resonance cone angle for a large part of the ray path. At Jupiter the magnetospheric densities are apparently much lower than those at Neptune, and the magnetic field strength is much larger. Consequently, the lower hybrid frequency will be larger than the whistler frequency for much of the ray path thereby resulting in an open index-of-refraction surface with no resonance cone.

It is clear from the table that source points near a longitude of 130° produce the largest values of T . For $\beta = 58^\circ$, $\lambda = 25^\circ$, $\Lambda =$

 TABLE 1. Selected Source Positions and Time Delays ($f = 10$ kHz)

β , deg	Source Position			T , s	Density Model A
	λ , deg	Λ , deg	L_{avg}		
50	-10	70	2.7	11.4	0
49	0	100	2.7	7.6	0
57	18	130	2.8	3.9	0
67	35	150	2.7	3.6	0
174	70	260	2.6	3.8	0
34	10	10	2.4	6.4	1.0
72	-20	30	1.8	1.6	1.0
49	0	100	2.7	15.5	1.0
53	10	120	2.9	9.4	1.0
58	25	130	2.8	61.6	1.0
61	25	140	2.8	35.1	1.0
67	35	150	2.7	23.3	1.0
93	60	180	2.7	4.6	1.0
103	65	190	2.6	4.9	1.0
123	70	210	2.4	8.3	1.0
174	70	260	2.5	12.0	1.0
85	49	330	2.2	6.0	1.0

130° we obtained $T = 61.6$ s. This is about $1/4$ of the value of $T \sim 250$ s reported by Gurnett et al. [1990]. In Figure 2 we show a three-dimensional plot of the ray path for this last case. Superimposed on the figure is the magnetic field line along which the source was located. The solid circle indicates the south magnetic pole. Clearly the ray path diverges substantially from the magnetic field line and extends to large L values. Two

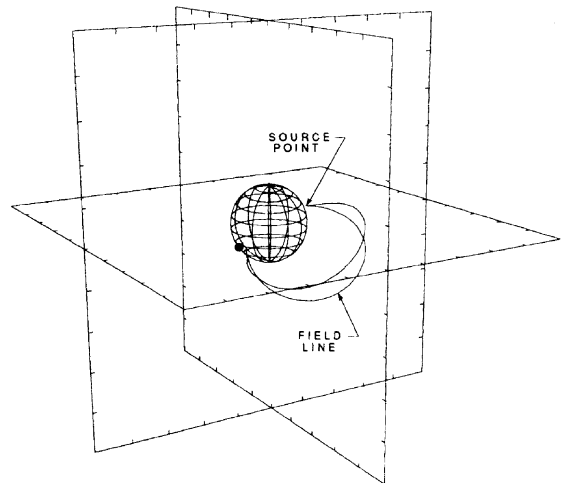


Fig. 2. A three-dimensional plot of the ray path of the whistler for a case where $T \sim 61.6$ s. The field line along which the source point was located is shown as well as the south magnetic pole (black dot).

perspective plots of the ray path in two dimensions are shown in Figure 3. The total change in magnetic longitude is about 10°. In Figure 4 we plot Ψ and Ψ_r versus ray path, s , and in Figure 5 we plot the index of refraction, N , and v_g/c (group velocity/speed of light) versus s . We may use these figures to understand a little more clearly the behavior of the whistlers in the ionosphere and magnetosphere of Neptune.

In Figure 4 we see that the wave normal angle, Ψ , is initially about 23° and that the wave propagates along the density gradient. The resonance cone angle is near 90° at this point. As the wave propagates out of the ionosphere, the wave normal angle initially decreases as the wave leaves the ionosphere, and then increases, rapidly approaching the resonance cone angle. In Figure 5 we see that the index of refraction initially decreases as the wave propagates out of the ionosphere but soon begins to

increase rapidly as Ψ approaches Ψ_r . As propagation proceeds into the magnetosphere and both f_p and f_c decrease, Ψ approaches Ψ_r and the magnitude of Ψ gradually increases and then decreases rapidly when Ψ is no longer close to Ψ_r . The wave again begins to approach Neptune and a new field line footprint in the opposite hemisphere. During this time both Ψ and N steadily decrease. As the wave enters the ionosphere for the second time near the conjugate footprint, at the south magnetic pole, Ψ_r increases to ~90°, and Ψ begins to increase. Because the south magnetic pole surface field strength is much larger than the north polar field strength, the gyrofrequency becomes relatively quite large, and f_p/f_c becomes less than 1 (Figure 6). The index of refraction reaches a minimum and begins to increase as f_p/f_c begins to increase, once again becoming greater than 1 well inside the ionosphere.

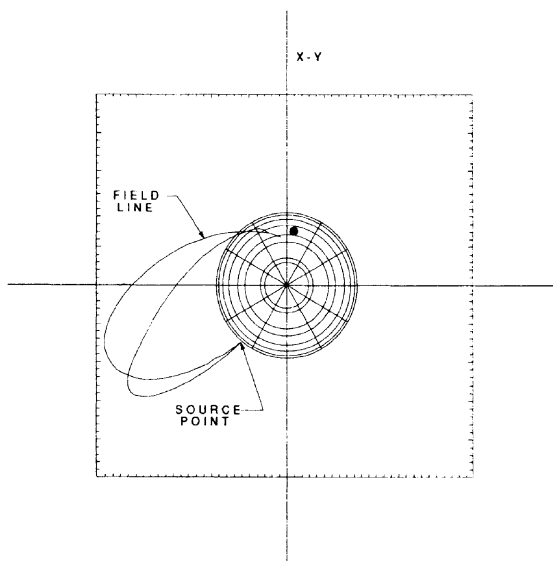


Fig. 3. Two perspectives in two dimensions of the same ray path shown in Figure 2. The top panel is the perspective from the north geographic pole.

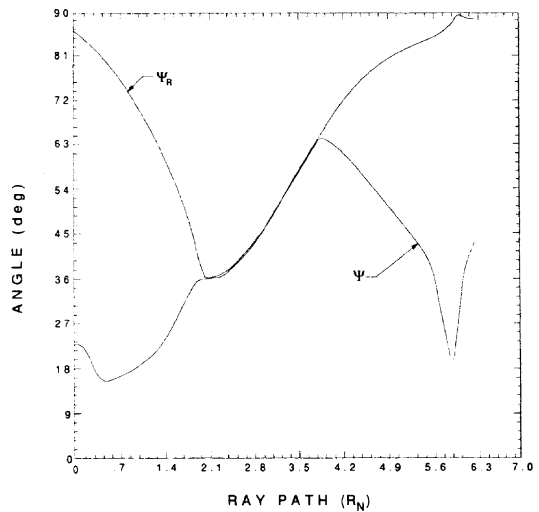


Fig. 4. The wave normal angle, Ψ , and the resonance cone angle, Ψ_r , versus ray path for the case with $T = 61.6$ s.

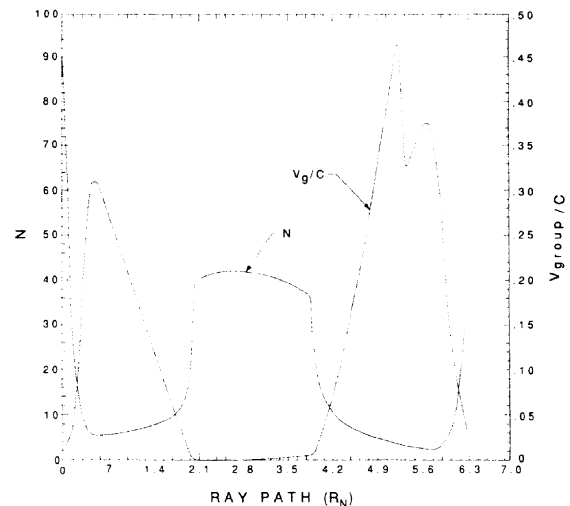
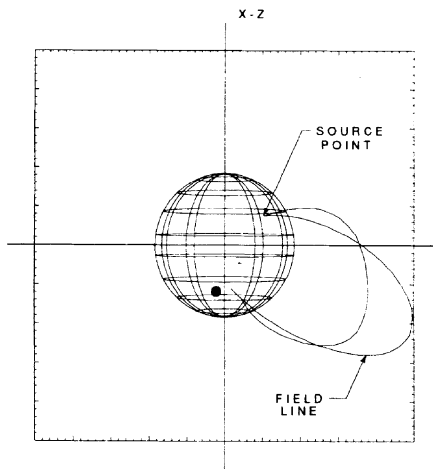


Fig. 5. The index of refraction and v_g/c versus ray path for the case with $T = 61.6$ s.

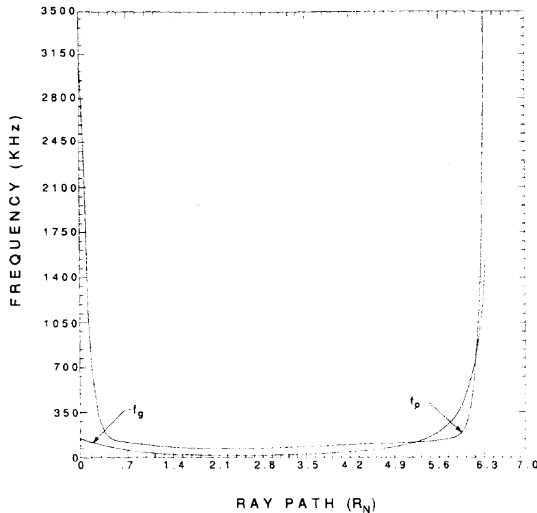


Fig. 6. Gyrofrequency and plasma frequency versus ray path for the case with $T = 61.6$ s.

CALCULATION OF DISPERSION

In order to estimate the dispersion of the rays we have evaluated the time delay for several frequencies for specific source points. The technique is tedious because, for a fixed source point in the ionosphere we require that the ray for each chosen frequency intercept a given observation point in the magnetosphere. The observation point was selected to be near the magnetic equator at a distance of $\sim 2.4 R_N$. This point was not an actual spacecraft position, but was chosen to be near sources which produced the largest dispersion. There are at least four parameters that could be adjusted at the source point to accomplish this: Ψ , β , λ , and Λ . In the limited time and finances available for this study, we were able to find ray paths at three frequencies that intercepted at the common point for whistlers with a source point at $\lambda = 25^\circ$, $\Lambda = 130^\circ$. We see from Table 1 that the total time delay for this source point at a frequency of 10 kHz is about 62 s.

To minimize the time required to obtain satisfactory results we limited the variable parameters to Ψ and β . We found interceptions for three frequencies with at most a 1° change in β . The initial value of the wave vector k varied by a maximum of 0.1° from the vertical. We display plots of $1/f$ versus T in Figure 7. The dispersion for this case is found from a least squares fit to be $D \sim 34,800$ s $\sqrt{\text{Hz}}$. The latter value is comparable to the value of $\sim 26,000$ s $\sqrt{\text{Hz}}$ obtained by Gurnett *et al.* [1990] for most of the whistlers. It is important to note that Gurnett *et al.* [1990] obtained D from the observations by measuring the slope of the curve of $1/f$ versus delay time. Their values are strictly determined by arrival time measurements as a function of frequency and are not dependent on the validity of equation (1). Our calculated values of D are obtained by essentially the same technique that was used to obtain the observed values.

SUMMARY AND DISCUSSION

We have shown that whistler propagation in the magnetosphere of Neptune leads to dispersions comparable to those observed by

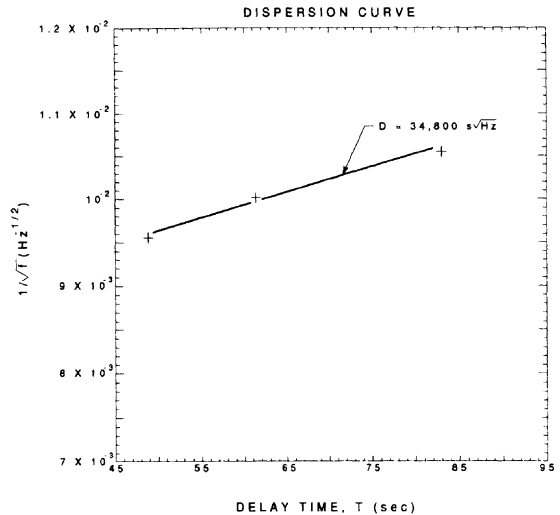


Fig. 7. $1/f$ versus delay time T for whistlers with a source at $\lambda = 25^\circ$, $\Lambda = 130^\circ$.

Voyager. The large dispersions are the result of propagation over large distances with wave normal angles very near the resonance cone angle. In the absence of damping, the waves initially propagate approximately along magnetic field lines and drift in magnetic longitude by about 10° per hop. The actual dispersion is clearly dependent on the density along the ray path, which is not known with certainty at this point. For a magnetospheric density model consistent with that suggested by Gurnett *et al.* [1990] we have obtained dispersions of $\sim 35,000$ s $\sqrt{\text{Hz}}$ for one-hop whistlers. This value of D is comparable to those observed by Gurnett *et al.* [1990] by the plasma wave instrument on board Voyager 2. Our results not only strongly suggest that the signatures observed by Gurnett *et al.* [1990] are in fact whistlers, but make the "requirement" of tens of multiple hops suggested by those authors unnecessary. However, there are still some discrepancies between our results and the observations of Gurnett *et al.* which we discuss below.

The observations indicate that all of the observed whistlers have approximately the same dispersion. Gurnett *et al.* [1990] suggested that this may be due to longitudinal drift of the ray path of the whistlers from a localized source. We suggest as an alternative to this explanation that only single-hop whistlers from a localized source may exist. Our results indicate that most of the dispersion has occurred by the time the wave has traveled to the magnetic equator. Thus a satellite observing whistlers from a localized source would see a relatively constant dispersion. This does not preclude the existence of multiple-hop whistlers, however.

Another factor that we must consider is the damping of the waves. We can obtain a crude estimate of the possibility of Landau damping by equating the local ionospheric thermal velocity to the parallel phase velocity of the whistler. We do not expect Landau damping outside the ionosphere because the plasma temperature would be much lower. For an ionospheric temperature of about $T = 950^\circ$ [Tyler *et al.*, 1989] we find the critical index of refraction for the onset of Landau damping is given by

$$N_{crit} \sim 2.5 \times 10^3 \cos(\Psi) \quad (8)$$

In the ionosphere of the opposite hemisphere, the ray of Figures 2 and 3 yields $\Psi \sim 42^\circ$, which yields $N_{\text{crit}} > 1500$ while the calculated value is $N \sim 25$. For all points along the ray path the largest wave normal angle of the wave is $\Psi \sim 63^\circ$ (Figure 4), which yields $N_{\text{crit}} > 1100$. The largest index of refraction of the wave for all points along the ray path is $N \sim 90$. We conclude therefore that no Landau damping is expected.

Most of the whistlers reported by Gurnett *et al.* [1990] were observed near the magnetic equator at a longitude of about 250° with $L < 2$. Our calculations indicate that the largest dispersions should result from whistlers with sources near geographic coordinates of $20^\circ < \lambda < 40^\circ$ and $130^\circ < \Lambda < 150^\circ$. For these sources the ray path intercepts a longitude of $\sim 170^\circ$ near the magnetic equator, and with $2.5 < L_{\text{avg}} < 3.0$. Attempts to make the model whistlers intercept the region of observation yielded a time delay of less than 10 s for one-hop whistlers.

From Figure 4 and Table 1 of Gurnett *et al.* [1990] the total travel time for the observed whistlers in the frequency range of $9 < f < 11$ kHz is seen to be between about 230 s and over 300 s. For all of the cases of this study the total travel time of the waves was less than 100 s, even though the model dispersions are comparable to those observed. This would indicate that the actual ray path or plasma parameters have not been exactly reproduced.

It is our belief that while in this initial study we have not been able to reproduce the observations exactly with regard to location we have shown that extremely large dispersions are to be expected for whistlers at Neptune. These large dispersions are primarily due to a large cold plasma density resulting in propagation near the resonance cone angle over great distances. By modifying the surface magnetic field model and/or the magnetospheric and ionospheric density model, it is quite possible to produce model whistlers that agree with the observations with respect to both location and dispersion. Such a study is most likely tedious, but may also provide an indirect measure of the surface magnetic field strength and cold plasma density, which at this point are still uncertain.

It is also not known at this time exactly what effect higher-order moments of the magnetic field might have on whistler dispersion. We are interested in determining what parameters control if or how rapidly Ψ approaches Ψ_r . At this time we know that higher magnetospheric density in combination with lower magnetic field strength increases the time delay. The initial angle of the wave with respect to the density gradient, the ionospheric and magnetospheric density, and the curvature and strength of the magnetic field all control how rapidly Ψ approaches Ψ_r . The relative importance of each is not well known at this time. We have found, however, that the ionospheric density appears to be the least sensitive of the parameters. We thus believe that magnetic field strength near the source and magnetospheric density are the main factors in producing large delay times and consequently large dispersions.

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