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MERGING OF AIRCRAFT VORTEX TRAILS: SIMILARITIES TO MAGNETIC FIELD MERGING

Donald A. Gurnett

Dept. of Physics and Astronomy, University of Iowa, Iowa City

Abstract. This paper discusses the phenomenological and formal similarities between the merging of aircraft vortex trails and the merging of magnetic field lines in a plasma. High-resolution photographs are shown of smoke trails from the wing tips of an airplane. These photographs show that the two vortex trails merge together downstream of the aircraft in a way similar to the merging of oppositely directed magnetic field lines in a plasma. Although there are some differences, this correspondence is apparently related to the fact that the vorticity equation in a fluid has the same mathematical form as the magnetic field equation in a MHD plasma. In both cases the merging proceeds at a rate considerably faster than would be predicted from classical estimates of the viscosity and resistivity. The enhanced merging rate in the fluid case appears to result from turbulence that increases the diffusion rate in the merging region.

INTRODUCTION

For many years the concept of magnetic field line merging, also called magnetic reconnection and field annihilation, has played a central role in space plasma physics. In this paper we describe a relatively little known fluid dynamics effect that appears to have a very close phenomenological similarity to magnetic merging. This phenomenon, which is the merging of the two vortex trails behind an airplane, can be readily seen in the condensation trail of a jet aircraft flying at high altitudes. As we shall describe, the aircraft vortex trail merging phenomenon provides a dramatic visualization of the merging process, and stimulates some interesting questions about the formal similarities between the two systems.

MAGNETIC FIELD MERGING

The concept of magnetic field line merging originated from suggestions by Giovanelli [1946] that magnetic neutral points could play an important role in the energy released by a solar flare. Neutral points, or in three dimensions, neutral lines, occur at the interface between plasma regions with oppositely directed magnetic fields. The magnetic field topology and plasma flow near such an interface have been discussed by numerous investigators, including Dungey [1953; 1961], McDonald [1954], Sweet [1958], Parker [1957], and Petschek [1964]. In the simplest two-dimensional case, the magnetic field lines form a separatrix in the form of an X, with the neutral point at the center of the X. The magnetic field is oppositely directed above and below, and to the right and left of the neutral point. Using the definition of Vasyliunas [1975], merging is said to have occurred if plasma flows across the separatrix. In the most general case, a two-dimensional geometry is not required, and a neutral line does not have to occur in the merging region.

To illustrate the processes involved in merging, Figure 1 shows a sketch of the X-type merging regions that occur in the Earth's magnetosphere. As first suggested by Dungey [1961] it is believed that magnetic field lines in the solar wind merge with the Earth's dipole magnetic field, thereby forming an X-type neutral point on the dayside of the Earth, as illustrated in Figure 1. After merging occurs the magnetic field lines are convected tailward over the polar cap by the motion of the solar wind. The direction of plasma flow is indicated by the arrows. The magnetic field lines over the polar cap are called "open" because they connect to the Earth at one end and the solar wind at the other. On the nightside of the Earth, these field lines convect toward a second neutral point in the magnetotail where merging again takes place. On the Earthward side of this neutral point, the field lines reconnect with their counterpart from the opposite hemisphere to form "closed" field lines, closed in the sense that they connect to the Earth at both ends. On the tailward side of the neutral point, the magnetic field topology can have a variety of forms, including closed-loop structures, with one or more neutral points located farther down the tail.

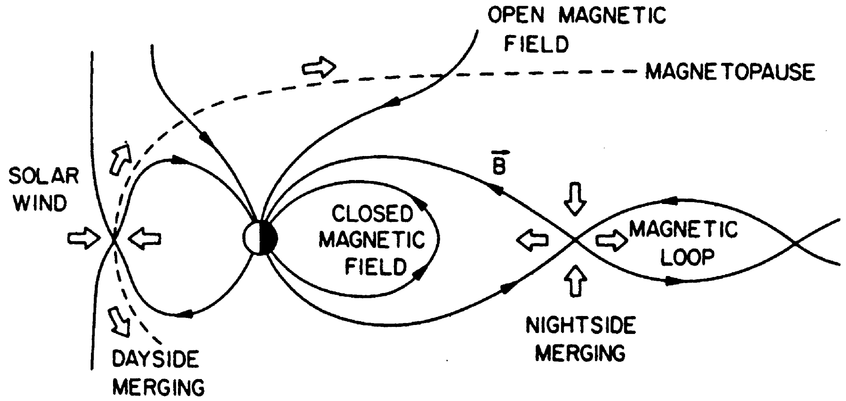

Fig. 1. Because of the solar wind flow past the Earth, interplanetary magnetic field lines merge with "closed" dipole field lines on the dayside of the Earth, convect over the polar cap, and then merge again with their southern hemisphere counterpart on the nightside of the Earth.

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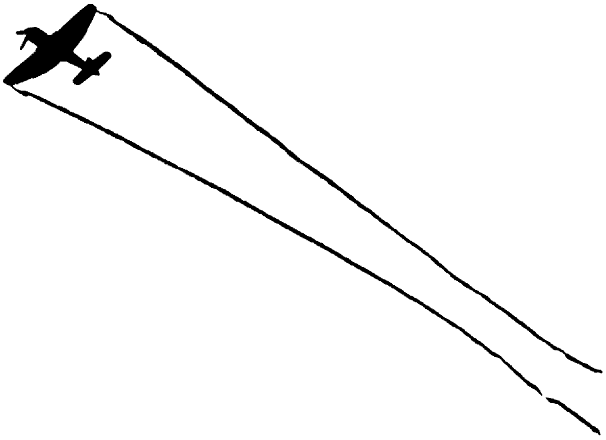

Fig. 2. Smoke injected into the airstream at the wing tip of an airplane reveals the presence of two counter-rotating vortex trails downstream of the aircraft. Wing tip injection produces a very tightly confined smoke trail along the central axis of the vortex.

For a recent discussion of merging in the Earth's magnetotail, see Hones [1985].

From very general considerations it can be shown that when merging occurs magnetic field energy is converted to plasma kinetic energy as the plasma flows across the separatrix. It is this energy conversion that makes merging of great importance in solar and magnetospheric physics. In the terrestrial magnetosphere plasma heating and energization caused by merging are believed to be responsible for the auroras that occur over the Earth's polar regions. For a discussion of these energy conversion processes see, for example, Vasyliunas [1975].

MERGING OF AIRCRAFT VORTEX TRAILS

From the very earliest studies of aircraft wakes, it has been known that an airplane moving through the air generates a coherent vortex system downstream of the wing. Although the flow pattern is quite complicated near the wing, in the region downstream of the aircraft the flow quickly organizes itself into two counter-rotating vortices aligned along the direction of motion, one extending backwards from each wing tip. The directions of rotation are such that a downward flow occurs in the region behind the wing. The downward flow is the direct result of the production of lift by the wing. For a review of these effects, see Batchelor [1967].

Usually the vortex trails behind an airplane are invisible. However, if smoke or condensation from an engine exhaust are injected into the airstream, the vortices can be seen. A particularly clear example of aircraft vortex trails is shown in Figure 2. In this case the vortices are made visible by the injection of smoke into the airstream at the tips of the wing. This injection arrangement provides a very narrow, dense smoke trail located almost exactly along the central axis of the vortex. Although it cannot be seen in this still photograph, a very rapid rotation of the smoke about the central axis can be easily seen with the naked eye. On the day that this photograph was taken, there was almost no wind or turbulence, so the vortex trails persisted for a particularly long time, about three to five minutes. The diameter of the smoke trail was a few tens of centimeters. For comparison, the wing span of the aircraft involved is about 12 meters.

The phenomenon of interest occurs about one to two minutes after passage of the aircraft. As time evolves, the two trails begin to distort and pinch together at certain points. Then rather suddenly, at one of these points the trails connect into an X-type configuration, and then separate into two U-shaped segments. This process is illustrated by the photograph in Figure 3, which was taken about 1 minute after the photograph in Figure 2, and about 5 seconds after the trails passed through the X-type configuration. Two distinct U-shaped structures can be seen, one to the right, and one to the left. The trails associated with the U-shaped structure to the left connect directly to the smoke generators at the tips of the wing. Near the left-hand side of the photograph, the two trails can be seen pinching together at a second point. A few seconds later the trails at this second point formed into an X-type configuration and then separated, leaving a closed loop to the right.

The same vortex merging process can be seen in the condensation trails of high flying jet aircraft. Because the engines on a jet aircraft are not located at the tips of the wings, the condensation trails are not as thin and well defined as in Figure 2. Nevertheless, the effect is easily seen. Figure 4 shows a condensation trail of a high flying commercial jet. Two X-type merging regions are clearly visible. After merging occurs, the trails form into a series of loops which gradually dissipated over a period of a few minutes. Vortex merging events of the type shown in Figure 4 can be seen almost anytime the atmospheric conditions are sufficiently steady for clearly defined

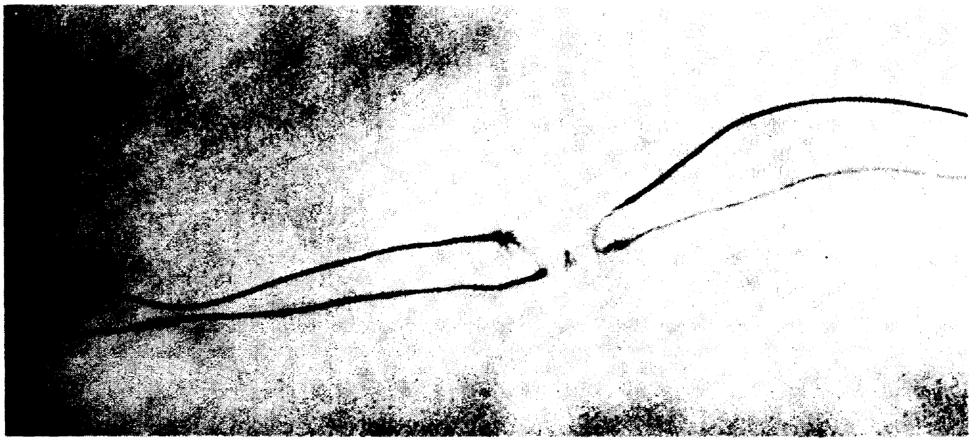

Fig. 3. A photograph taken shortly after Figure 2 showing the merging of the two vortex trails. The trails passed through the X configuration about five seconds before this photograph was taken. A second merging region is about to form on the left side of the photograph.

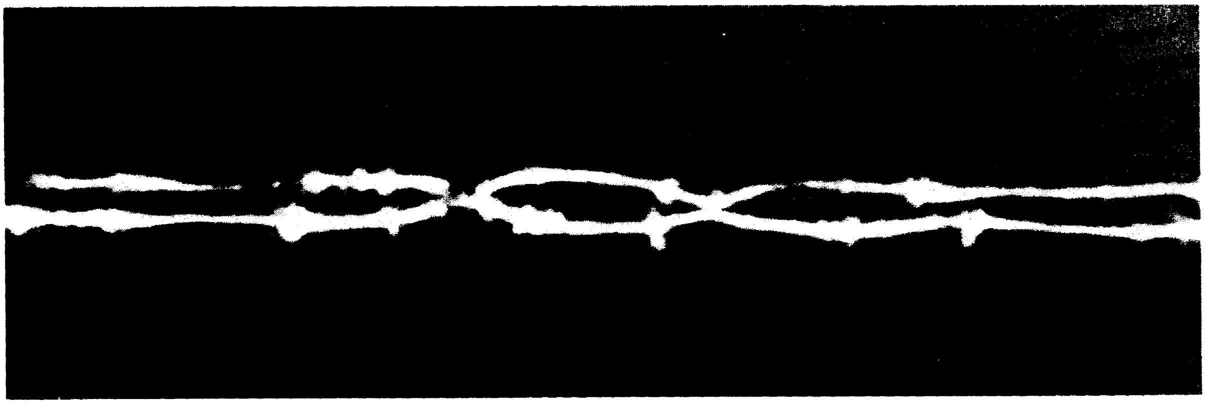

Fig. 4. A photograph of a high flying commercial jet showing the formation of several X-type merging regions in the downstream condensation trial. This process can be seen almost any time that atmospheric conditions are sufficiently stable for the formation of well-defined condensation trails.

condensation trails to form. The aircraft must have engines on the wing in order to produce two well-separated condensation trails.

The vortex merging results presented above are not basically new. The effect is well-known to aircraft pilots, although very few photographs of the effect have ever been published, particularly ones with resolution comparable to that of Figure 2. In the fluid dynamics literature, the phenomenon is known as the Crow instability, after Crow [1970] who first analyzed the linear growth phase of the perturbations in the trails. A series of photographs, comparable to Figure 4, showing the formation of X lines and loops in the vortex trails of a high flying jet aircraft is given in "An Album of Fluid Motions" by Van Dyke [1982]. These photographs were obtained from an earlier work by Smith and Beesmer [1959]. Although the initial growth of the perturbations in the vortex trails is well understood, the dynamical processes involved in the merging and formation of loops is the subject of continuing discussion and analysis. In the fluid dynamics literature the process that we refer to as merging is sometimes referred to as the "coalescence instability." For a recent work on this subject, see for example, Pumir and Sigga [1987].

THE FORMAL SIMILARITY

The phenomenological similarity between magnetic merging and the merging of aircraft vortex trails suggests a close underlying formal similarity. Indeed this appears to be the case. In magnetohydrodynamics (MHD) it has been known for many years (see, for example, Alfvén and Fälthammar [1963]) that the time rate of change of the magnetic field, $\vec{B}$, is given by

$$\frac{\partial\vec{B}}{\partial t} = \nabla\mathrm{x}(\vec{u}\mathrm{x}\vec{B}) + \frac{\eta}{\mu_0}\nabla^2\vec{B} \quad , \qquad (1)$$

where $\vec{u}$ is the fluid velocity and η is the resistivity. The first term on the right represents the change caused by convection, and the second term represents the change caused by diffusion. In the limit of small resistivity or large scale sizes, the convection term dominates the diffusion term. This limit leads to the well-known "frozen-in-field" condition in which the magnetic field lines are carried by the fluid. For an Ohm's law type of conductivity, magnetic merging does not occur in the limit of zero resistivity [Sweet, 1958]. Therefore, for merging to occur a finite resistivity is required. For other factors that can affect the merging rate, see the review by Vasyliunas [1975].

In fluid dynamics the vorticity is defined as the curl of the flow velocity, $\vec{\omega} = \vec{\nabla} \times \vec{u}$. It is easy to show that vorticity lines have zero divergence ($\vec{\nabla}\cdot\vec{\omega} = 0$), just as magnetic field lines have zero divergence ($\vec{\nabla}\cdot\vec{B} = 0$). Under very general conditions it can be shown (see, for example, Batchelor [1967]) that the time rate of change of the vorticity is given by

$$\frac{\partial\vec{\omega}}{\partial t} = \vec{\nabla}\mathrm{x}(\vec{u}\mathrm{x}\vec{\omega}) + \nu\nabla^2\vec{\omega} \quad , \qquad (2)$$

where ν is the kinematic viscosity. As can be seen, this equation is identical to the magnetic field equation, with the vorticity playing the same role as the magnetic field. Therefore, if the velocity fields are the same, vortex lines in a fluid would evolve in the same way that magnetic field lines evolve in a MHD plasma. This formal similarity has been recognized by several authors, including, for example, Batchelor [1950] and Axford [1984].

In the case of an airplane moving through the atmosphere, the viscosity of the air is sufficiently small that the vorticity lines are to a very good approximation "frozen in" the fluid. Smoke injected along the axis of the vortex, as in Figure 2, then traces out a line of vorticity. The basic geometry involved is illustrated in Figure 5. Although the flow pattern behind an

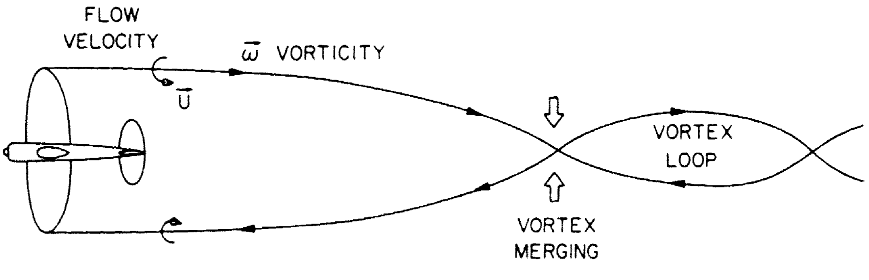

Fig. 5. A schematic diagram showing the merging of vortex lines, $\vec{\omega} = \vec{\nabla} \times \vec{u}$, downstream of an aircraft. The vortex lines in a fluid obey the same equation of motion as magnetic field lines in a MHD plasma.

aircraft leads to oppositely directed vortex lines in the downstream region, just as for magnetic field lines in the Earth's magnetotail, there are clearly some differences in detail. In the aircraft wake, the vortex lines are concentrated mainly in two vortex tubes that are estimated to have diameters of to 20% of the wingspan [Crow, 1970], whereas magnetic field lines completely fill the two lobes of the magnetotail. Therefore, merging of vortex lines behind an aircraft does not occur until the the two flux tubes come into contact, whereas merging in the Earth's magnetotail occurs continuously, since oppositely directed magnetic fields are always present on opposite sides of the neutral line. A non-zero viscosity is required for vortex merging to occur in a fluid, just as a non-zero resistivity is required for magnetic merging to occur in a MHD plasma.

COMMENTS

Having noted the phenomenological and formal similarities between magnetic field line merging in a plasma and vortex merging in a fluid, we next discuss the limits of these similarities and consider what could be learned by comparing these two areas of research. Although Equations 1 and 2 for the magnetic field and vorticity have the same mathematical form it is quite clear that the full set of fluid equations and the full set of MHD equations represent two completely different systems. Some of these differences are immediately apparent. In Equation 2, the vorticity is directly related to the flow velocity, $\vec{\omega} = \vec{\nabla} \times \vec{u}$, whereas in Equation 1 the magnetic field is related to the flow velocity via a more complex system of equations involving Ohm's law and Maxwell's equations. Therefore, although both $\vec{\omega}$ and $\vec{B}$ evolve in the same way for a given velocity field, there is no guarantee that the velocity fields will be the same. In general, they will not be the same. In a fluid there is no analog to magnetic field stress in an MHD fluid, so the fluid velocity does not evolve in exactly the same way. The best that can be expected is that the flow patterns in the merging region may be qualitatively similar.

In magnetic merging the extremely low collision frequencies typically encountered in space plasmas leads to resistivities too low to account for the observed merging rates. It is interesting to note that a similar problem exists for vortex merging. In his analysis, Crow [1970] comments that the vortex merging proceeds at a much higher rate than would be expected from simple diffusion. In space plasmas, the low resistivity has led to numerous suggestions for increasing the merging rate, including, for example, anomalous resistivity caused by micro-instabilities [Piddington, 1967], ion-tearing mode effects [Schindler, 1974] and ion inertial effects [Coroniti, 1985]. In the vortex merging case, we believe that it is worth pointing out that highly turbulent micro-instabilities can be seen disrupting the vortex in the vicinity of the X point. In Figure 3 this turbulence leaves a small cloud of smoke, no longer entrained in the vortex, at the point where the X occurred. The turbulence also produces a pronounced thinning of the smoke trail around the corner's of the U's. Therefore, it seems likely that turbulence plays an important role in the vortex merging process by increasing the diffusive transport rate, effectively producing an "anomalous viscosity" in the merging region. This turbulence is highly reminiscent of the turbulent magnetic fields found near the merging region in the terrestrial magnetotail [Coroniti et al., 1977]. In the fluid case, the turbulence is probably caused by large velocity shears that occur as the two vorticities pinch together.

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D. A. Gurnett, Dept. of Physics and Astronomy, University of Iowa, Iowa City, IA 52242.

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