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THE CONDUCTANCE OF AURORAL MAGNETIC FIELD LINES

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Abstract. Recent results from the Dynamics Explorer satellites have indicated that in the auroral zone a linear relationship exists between the field aligned current density and the potential drop parallel to the magnetic field lines. Evidence for this "Ohm's law" relationship was found in the mapping of perpendicular electric fields and field-aligned currents between high and low altitudes. The mapping depends on the perpendicular wavelength of the electric field variations. A scale length in the mapping formula is determined by the ratio of the parallel field line conductance and the ionospheric Pedersen conductance. The wavelength and the conductivity ratio also control the relationship between the perpendicular electric and magnetic fields at high altitudes.

We show here that at the short-wavelength limit the ionospheric conductivity is no longer important in the relationship between the north-south electric field and the east-west magnetic field at high altitudes (i.e., above the parallel potential drop). At the short-wavelength limit the relationship takes on a simple form: The integral of the perpendicular electric field results in a potential profile which, according to the linear theory, is proportional to the current density. Assuming that the currents are in the form of "infinite sheets" orientated east-west, the second integral of the electric field is proportional to the magnetic field.

High time-resolution data from the DE-1 satellite are shown here for two events with very large electric fields which reversed directions within a short distance. The results agree very well with the linear theory. The field line conductance is determined to be of the order of 10^{-9} mho/m^2. The same conductance appears to be valid for both upward and downward currents. Ions are accelerated from the ionosphere to magnetosphere by the potential drops in regions of upward current.

Introduction

A considerable body of scientific evidence exists which indicates that there are electric fields parallel to the magnetic field lines in the auroral zone. These parallel electric fields accelerate electrons into the ionosphere to form the aurora, and they also accelerate ions in the opposite direction. It is not so well known exactly how the parallel electric fields are formed. Several mechanisms have been suggested, such as the magnetic mirror force, double layers, anomalous resistivity, and hydromagnetic waves. These theories are discussed in the reviews by Shawhan et al. [1978] and Stern [1983].

In order to determine which mechanism is responsible for the parallel electric fields it is useful to study the current-voltage relationship along the magnetic field lines. Lyons et al. [1979] and Menietti and Burch [1981] had found evidence for a linear "Ohm's law" relationship between the field-aligned current and potential drop, based on measurements of precipitating electron energy fluxes at the ionosphere. The "Ohm's law" relationship was also verified in a recent study of the perpendicular electric fields measured nearly simultaneously by the two Dynamics Explorer spacecraft near magnetic conjunctions in the auroral zone [Weimer et al., 1985]. Parallel field line conductances of the order of 10^{-8} mho/m^2 were measured.

The study by Weimer et al. [1985] had been limited to electric field and current structures with spatial widths greater than 20 km at the base of the field lines. Naturally, the question arises about whether or not the same linear relationship still holds for very narrow current sheets. The 20 km limit was imposed by the measurement of the electric field on the high altitude DE-1 satellite. The electric field in the orbit plane of DE-1 is measured with a single, rotating double-probe. A static electric field appears in the "raw" data as a sine wave with a six second period. In the previous study the electric fields were derived from a least square fit of the raw data to a sine wave. This procedure works extremely well as long as the electric fields change on a time scale greater than the satellite spin period, but rapid, short-wavelength variations are filtered out. In order to obtain the electric field with a better spatial resolution, the sine wave modulation can be removed from the data in a more direct manner. In this paper we will show electric and magnetic field measurements with the highest possible resolution. The data shows that an "Ohm's law" is still valid for current structures just a few km across.

Theory

In a paper by Smiddy et al. [1980] it is shown that the height-integrated ionospheric Pedersen conductivity (Σ_P) is an important factor in the relationship between the field-aligned currents and the perpendicular electric fields. Just above the ionosphere the relationship is:

$$j_{\parallel} = -\frac{dE_x}{dx} \Sigma_P \tag{1}$$

Here we use a coordinate system in which the Z axis is upward along the magnetic field line, X is southward, and Y is eastward. The field-aligned currents are not measured directly but are inferred from the derivative of the east-west magnetic field component measured on the satellite while moving in the north-south direction through sheets of current, which are assumed to be very long in the east-west and up-down directions. The magnetic and electric fields are related to each other according to

$$\Delta B_y = -\mu_o \Sigma_P E_x \tag{2}$$

where ΔB_y is the difference between the measured magnetic field and the earth's dipole field. Equation (2) has been confirmed by a number of measurements of magnetic and electric fields just above the ionosphere [Smiddy et al. 1980; Sugiura, 1984]. At higher altitudes Equation (2) may not be valid due to the possible existence of an electric potential drop parallel to the magnetic field ($V_{\parallel}$). The "Ohm's law" we are investigating assumes that the field-aligned current density at the ionosphere and the parallel potential drop are related by

$$j_{\parallel} = -aV_{\parallel} \tag{3}$$

where "a" is the field line conductance. The sign convention is such that the current is positive (upward) when the potential drop from low to high altitude is negative. The physical justification for Equation (3) has been discussed by Knight [1973], Lyons et al. [1979], Fridman and Lemaire [1980], and Chiu et al. [1981].

It is shown by Weimer et al. [1985] that with the definition of a constant inverse scale length or "critical wavenumber",

$$\frac{a}{\Sigma_P} = k_o^2 \tag{4}$$

the Fourier transforms of the high-altitude quantities $j_{\parallel}$, B_y, and E_x have a relationship which depends on their spatial wavenumber, k :

$$\tilde{j}_{\parallel} = \frac{-i\,k}{(k/k_o)^2 + 1} \Sigma_P \tilde{E}_x \tag{5}$$

$$\mu_o \tilde{j}_{\parallel} = i\,k\,\tilde{B}_y \tag{6}$$

The tildas indicates that the quantities are transformed from a spatial domain to a wavenumber domain. The spatial variations are presumed to be in the north-south (x) direction.

With a field line conductance of 10^{-8} mho/m^2 and a Pedersen conductance of 5 to 10 mho, the inverse of the critical wavenumber, k_o^{-1}, will be in the range of 22 to 32 km. The wavelength $\lambda_o = 2\pi/k_o$ is 140 to 200 km. At the limit of a very small wavelength and large wavenumber, where $(k/k_o)^2 \gg 1$, Equation (5) can be simplified to

$$\tilde{j}_{\parallel} = -(i/k)\,a\,\tilde{E}_x \tag{7}$$

From Equation (3) we see that

$$-ik\tilde{V}_{\parallel} = \tilde{E}_x \tag{8a}$$

or in real space,

$$-\frac{dV_{\parallel}}{dx} = E_x \tag{8b}$$

where E_x is the measured electric field at the satellite altitude. Therefore, at very small wavelengths the ionospheric conductivity does not influence the high altitude electric field, and there is a very simple relationship between the north-south electric field and the parallel potential drop. By integrating E_x one can obtain a "potential profile" ($V_{\parallel}$ as a function of x), which should be proportional to the current density if the "Ohm's law" of equation (3) is correct. Integration of E_x a second time (with multiplication by 'a' and μ_o) should result in ΔB_y. This prediction can be tested by comparing E_x and ΔB_y in cases where the short-wavelength variations are large in comparison to the long-wavelength variations.

Observations

An example of high-resolution electric field data from the 200 meter double probe on DE-1 [Shawhan et al., 1981] is shown in the top panel

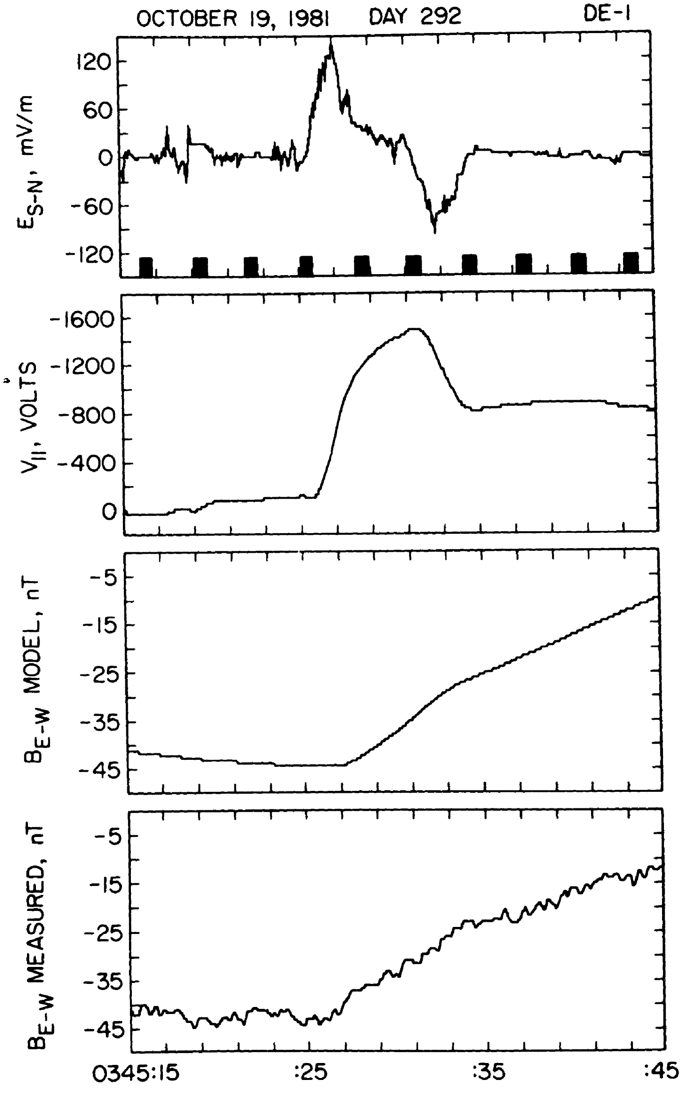

Fig. 1. Top graph is north-south electric field measured in the orbit plane of DE-1 on day 292 of 1981. The dark blocks above the time axis indicate where data gaps have been filled in. The second graph shows the potential obtained by integrating this electric field, and the third graph shows a model of the east-west magnetic field obtained with a second integration. At the bottom is shown the measured magnetic field.

of Figure 1. The "raw" electric field (which is spin modulated) is digitized at a rate of 16 samples per second. The electric field in Figure 1 has had the spin modulation removed by a technique in which each measurement is divided by the sine of the angle between the double-probe antenna and the magnetic field in the spin plane. This works well when the "spin phase angle" is large, but when the antenna is nearly parallel to the magnetic field there can be problems: A small error in the phase angle results in a distorted electric field, which goes to infinity when the angle is zero. To eliminate this effect gaps are introduced in the de-spun data whenever the magnitude of the phase angle is less than 30 degrees. The subsequent steps in the data analysis, however, require a continuous electric field, so the gaps are filled in with values which smoothly connect the data on both sides of the gaps. The resulting plot shows a large amplitude electric field which points southward for 6.0 seconds then reverses to a northward direction for 3.5 second. At the time of the measurement the velocity component of DE-1 perpendicular to the magnetic field was 4.77 km/sec (southward), thus the total width of this structure is 45 km. These fields were measured at an altitude of 10,500 km and invariant latitude of 69.4°, so the 45 km "maps" to a width of 9.5 km at the base of the magnetic field line.

Integration of the electric field plot in Figure 1 (also multiplied by the appropriate time and velocity constants) results in the electric potential function which is shown in the second plot from the top in the same figure. Multiplication of this potential by a conductance results in a value for the local current density; integration a second time results in the east-west deviation of the magnetic field shown in the third plot. The bottom plot in Figure 1 shows the actual value of the magnetic field measured by the magnetometer on DE-1 [Farthing et al., 1981].

As can be seen there is a very good agreement between the integrated and measured values. To obtain this agreement a trial-and-error adjustment of two parameters was necessary, namely a voltage offset and the field-line conductance. The potential in Figure 1 is shown to start at zero. A constant voltage may be added to compensate for the actual initial value.

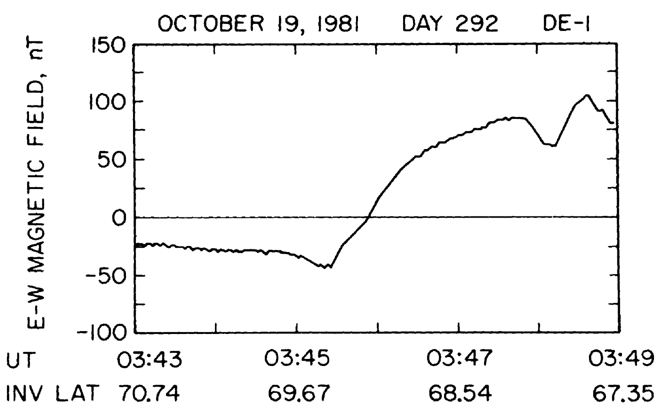

Fig. 2. East-west magnetic field component from the magnetometer on DE-1 from 3:43 to 3:49 UT on day 292, 1981. The earth's dipole field has been subtracted from the measured values.

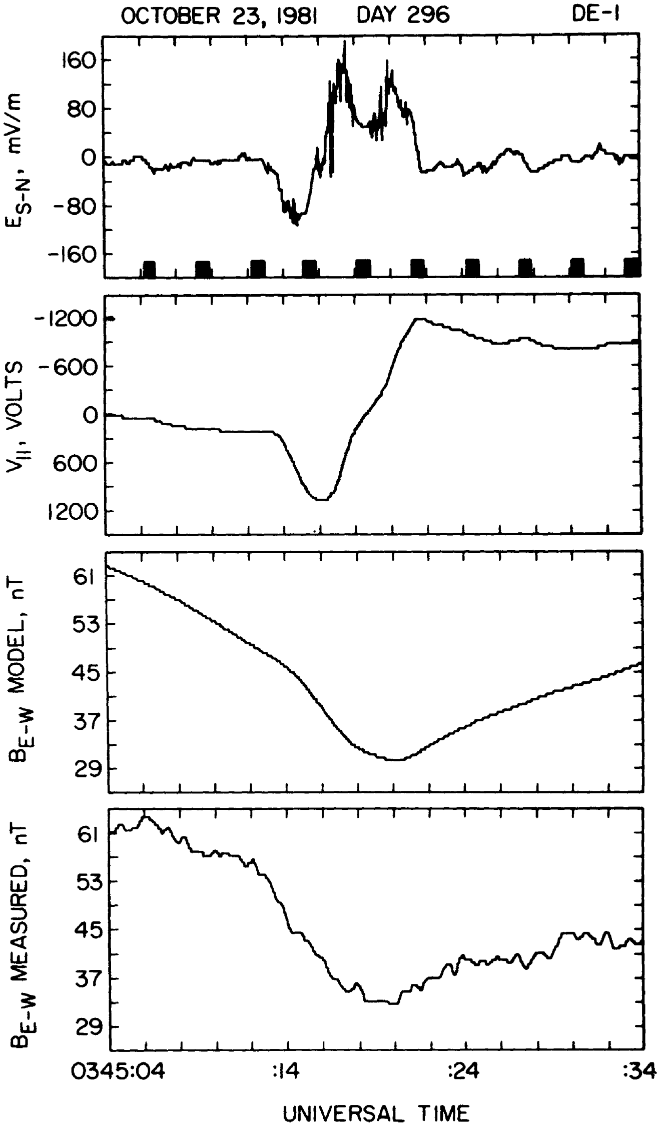

Fig. 3. Electric and magnetic fields measured with DE-1 on day 296, 1981. The format is the same as in Figure 1. The third graph from the top was obtained by twice integrating the measured electric field, while the measured magnetic field is shown at the bottom.

An offset of +200 volts yielded the best result. This positive potential at the high altitude location corresponds to a downward current. The justification for this assumption is given in Figure 2, which shows the measured east-west magnetic field for a much longer period of time. Prior to the electric field spike at 3:45:25 UT there was a gradual negative (westward) slope in the magnetic field, which corresponds to a downward current. The large electric field occurs at the boundary between the downward and the following upward current. A narrow but large magnitude upward current sheet is located right on the boundary. This peak in the upward current occurs precisely at the point where the parallel potential is most negative.

The addition of a constant to the parallel potential corresponds to the addition of a constant slope in the model magnetic field. The magnitudes of the relative changes in the magnetic field are determined by the value chosen for the field line conductance. In this case a conductance of $4 \cdot 10^{-10}$ mho/m^2 yielded the best match between the model and measured magnetic field. At the point where the potential drop is -1300 volts the local current density is $5.2 \cdot 10^{-7}$ A/m^2. Due to convergence of the magnetic flux tubes both the current density and the east-west magnetic field increase towards lower altitudes. The "mapping" factor for both of these quantities from a high-altitude location to the base of the magnetic field line is $L^{3/2}\cos^3\lambda_m$, where λ_m is the magnetic latitude. For the present case the mapping factor is 4.32, so the "normalized" field line conductance is $1.7 \cdot 10^{-9}$ mho/m^2. This is the ratio between the current density at the base of the field line ($2.2 \cdot 10^{-6}$ A/m^2) and the total potential drop along the field line ($V_{||}$=1.3 kV).

Another case is shown in Figure 3. This data is from day 296 of 1981. The format is the same as in Figure 1. This event has some similarities to the previous case, yet there are important differences. Figure 4 shows that, like before, the electric field "spike" occured on the southward edge of a region of downward current. But this time the downward current is much larger in magnitude and confined to a relatively narrow region, embedded within a larger region of upward current. Whereas in the previous case most of the magnetic field variations were due to upward currents, in this second example the downward currents are equally important.

As before, different parameters were tested in order to find a model magnetic field which best

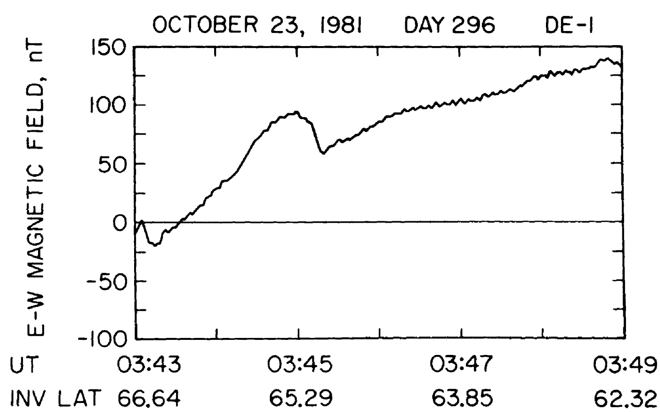

Fig. 4. East-west magnetic field measured with DE-1 from 3:43 to 3:49 UT on day 296, 1981. The geomagnetic field has been removed from the data.

matched the measured field. In this case the most reasonable results were obtained with the addition of a +3 mV/m offset to the electric field before doing the first integration. This compensated for a negative, large scale convection electric field which would be linearly proportional to ΔB according to Equation 2. A +500 volt offset was added to the potential before doing the second integration. This is justifiable on the basis that the region of downward current had been penetrated a few seconds before the starting time of Figure 3. As before, a conductance of $4 \cdot 10^{-10}$ mho/m^2 was used to generate the model magnetic field. The DE-1 satellite was at an altitude of 9000 km and an invariant latitude of 65.1°, so the current projection factor to the field line base is 3.75. This results in a normalized field line conductance of $1.5 \cdot 10^{-9}$ mho/m^2.

Discussion

The evidence which has been shown here provides a convincing case for the validity of Equation 3 for current structures with small spatial widths. The linear relationship between current density and potential is the simplest and, as far as we know, only explanation for the magnetic field signatures which are observed concurrent with large-amplitude electric fields. In general, the large amplitude electric fields occur on the boundary between upward and downward currents, where the high-altitude potential must reverse signs within a short distance.

A certain amount of adjustment of the starting potential and field-aligned conductance was required to get the good agreements shown here. But the adjustments were within reason and relatively minor. The second integral would not have given the observed agreement if the "Ohm's law" was not a good approximation. Indeed, given the gaps in the measured electric field data it is amazing that the integrated values come as close as they do to matching the measured magnetic fields, since the integrations cause errors to accumulate forward in time.

The field line conductances measured with this technique generally are close to 10^{-9} mho/m^2. The question remains: which of the proposed mechanisms for creating a parallel potential drop along magnetic field lines can produce a conductance of this value? It is important to note that the conductance is the same for both upward and downward currents. Although a better model for the data in Figure 3 maybe could be obtained by using different conductances for the upward and downward current regions, the values can not be much different. This fact tends to eliminate the magnetic mirror force as a cause of the parallel potential drop, since it works in one direction only. It also seems that the kinetic theory yields smaller values for 'a' [Fridman and Lemaire, 1980] although the difference may be within limits of the model. In both examples shown here intense, short-wavelength "turbulence" in the electric field is present where the currents are most intense, suggesting that large amplitude plasma waves may be present. Thus anomalous resistivity [Lysak and Dum, 1983] may be important. We emphasize again that the results presented here are restricted to the domain of small perpendicular scale lengths.

The data suggests that there is a current generator in the magnetosphere above the observation point—this current generator may have an associated electric field, depending on the source impedance [Lysak, 1985]. The Kelvin Helmholtz instability may be the source of these currents (P. F. Bythrow, unpublished manuscript, 1985). It appears that the currents produce the parallel potential drops, perhaps by the formation of double layers or anomalous resistivity. We note that the "Ohm's law" also suggests that $\bar{J} \cdot \bar{E} > 0$ above the ionosphere and that electromagnetic energy is dissipated through ion acceleration in the magnetosphere.

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