ELECTRON ACCELERATION BY LANDAU RESONANCE WITH WHISTLER MODE WAVE PACKETS

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Abstract. Recent observations of electrostatic waves associated with whistler mode chorus emissions provide evidence that electrons are being trapped by Landau resonance interactions with the chorus. In this paper we discuss the trapping, acceleration and escape of electrons in Landau resonance with a whistler mode wave packet. We show that acceleration can occur by both inhomogeneous and dispersive effects. The maximum energy gained is controlled by the points where trapping and escape occur. Large energy changes are possible if the frequency of the wave packet or the magnetic field strength increase between the trapping and escape points. Various trapping and escape mechanisms are discussed.

Introduction

Plasma wave observations from the ISEE-1 spacecraft have recently shown that whistler mode chorus emissions are often accompanied by bursts of electrostatic noise near the electron plasma frequency [Reinleitner et al., 1982]. An example of this relationship is illustrated in Figure 1 which shows a band of chorus between about 200 to 300 Hz and the associated electrostatic bursts from about 5 to 8 kHz, slightly below the electron plasma frequency, f_p. The electrostatic bursts often occur on a one-to-one basis with discrete "hook-shaped" features in the chorus band. In these cases the amplitude of the electrostatic bursts shows a well defined amplitude modulation at the frequency of the chorus emission. This modulation effect is illustrated in Figure 2, which shows simultaneous measurements of the electric field of the electrostatic noise and the chorus, appropriately filtered to reveal the waveform of each signal. The electrostatic noise is seen to consist of a nearly monochromatic emission modulated in phase with the chorus signal. The bandwidth of the bursts in Figure 1 appears to be much broader because of the modulation by the chorus wave. Polarization measurements show that the electric field of the electrostatic wave is aligned approximately parallel to the static magnetic field. The chorus has both electric and magnetic components, as expected for the whistler-mode of propagation. Typical amplitudes are about 50 μV/m for the electric field of the electrostatic bursts and 300 μV/m and 40 pT for the electric and magnetic fields of the chorus.

A theoretical model to explain the main features of the above interaction has been proposed by Reinleitner et al. [1983] and has been confirmed by Matsumoto et al. [1983] using a computer simulation. This model assumes that electrons are trapped by a Landau resonance interaction with the chorus and phase bunched to produce a beam that excites the electrostatic waves. Trapping by Landau resonance interactions at the phase velocity, $v_p = \omega/k_\parallel$, of whistler mode waves has been considered by several investigators, including for example, the early work of Gallet [1959] and the more recent studies by Nunn [1971; 1973], and Tkalcevic [1982]. For trapping to occur the chorus must be propagating at an oblique angle to the magnetic field so that the wave has an electric field component $E_\parallel$ along the magnetic field. Once trapped the electrons are spatially bunched near the minimum in the effective potential well produced by the parallel electric field of the chorus. The spatial bunching of the trapped electron "beam" explains why the electrostatic noise occurs in discrete bursts in phase with the chorus signal, as in Figure 2. Each bunch of trapped electrons excites a burst of electrostatic noise via a two-stream instability.

The identification of the mode of the electrostatic noise has been somewhat difficult. The electrostatic waves cannot be simply the Langmuir oscillation at $\omega^2 = \omega_p^2 + (3\kappa T/m)k^2$, because the observed oscillation frequency ω is below the electron plasma frequency ω_p. The downward frequency shift can be explained if the beam velocity is comparable to the thermal velocity of the background plasma [Briggs, 1964]. Comparisons of the chorus phase velocity with the thermal velocity of the plasma [Reinleitner et al., 1983] show that the thermal correction produces a downward frequency shift that is in reasonable agreement with the observations. The electron beam density can be estimated from the growth time of the electrostatic waves, which from Figure 2 is seen to be about 0.5 msec. These estimates indicate that the ratio of the electron beam density to the ambient electron density is about 10^{-3}.

Direct observations of the beam produced by the trapped electrons is difficult because the chorus emissions last only about 1 second, whereas it takes tens of seconds to minutes for a typical plasma instrument to collect an electron distribution function. Observations by Reinleitner et al. [1983] using the LEPEDEA plasma analyzer of Frank et al. [1978] show that beam-like enhancements in the field-aligned electron fluxes are observed in association with the electrostatic bursts. These beam-like enhancements occur over a range of energies, typically 400 to 600 eV, that are near the Landau resonance energy of the chorus wave. Since the beam-like enhancements are presumably produced by acceleration of electrons from the thermal part of the distribution, the question naturally arises as to how this acceleration occurs. The purpose of this paper is to discuss possible Landau acceleration processes and to consider mechanisms by which electrons can be trapped and can escape from whistler mode wave packets.

Dispersive Acceleration

Once an electron has been trapped in the potential well any change in the phase velocity of the wave can cause the particle to accelerate. An obvious possibility for accelerating electrons trapped by the Landau resonance interaction with chorus is to change the phase velocity as the wave propagates away from the equator toward the Earth. This mechanism relies on plasma inhomogeneities and was discussed by Reinleitner et al. [1983]. We would now like to describe another mechanism, which we call dispersive acceleration, that can occur whenever a coherent wave packet of finite bandwidth propagates through a dispersive medium. This mechanism can accelerate trapped particles even when the medium is completely homogeneous.

To illustrate the principles involved we first consider a special case, which has to do with the dispersive propagation of surface waves on water. For deep water the dispersion relation is $\omega = \sqrt{gk}$, where g is the acceleration of gravity. A classic problem, first considered by Cauchy in 1815, is to consider the asymptotic form of the waves produced by dropping a stone in water (impulsive disturbance at t = 0). The solution to this problem can be obtained by using the method of stationary phase, and is given by the real part of

$$f(x,t) = \left(\frac{gt^2}{4\pi x^3}\right)^{1/2} \exp\left[i\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right)\right] \quad , \qquad (1)$$

Paper number 3L0867.
0094-8276/83/003L-0867$03.00

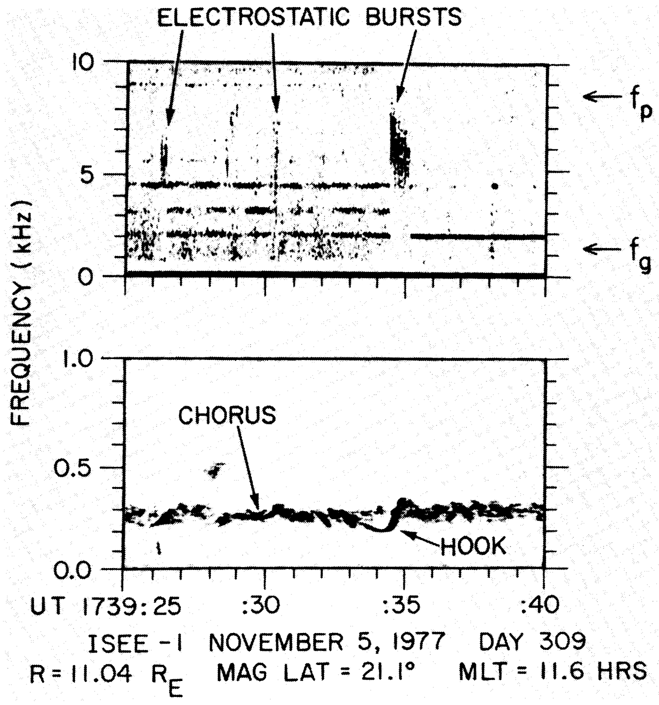

Fig. 1. An example of whistler mode chorus emissions (lower panel) and the associated electrostatic bursts (upper panel). The electrostatic bursts are often closely associated with hook-shaped features in the chorus band, usually occurring on the rising part of the hook.

where x is the distance from the source [Lamb, 1932]. Consider now a particle (surf board rider) trapped at a potential minimum of this wave. The particle moves at a point of constant phase, ϕ, which can be written

$$\frac{gt^2}{4x} = \phi \quad . \tag{2}$$

The velocity of this point is

$$v = \frac{dx}{dt} = \frac{1}{2}\,\frac{gt}{\phi} \quad . \tag{3}$$

As can be seen, the velocity increases linearly with time, thereby producing a constant acceleration. The existence of this acceleration was recognized by Lamb [1932] and others and is one of the remarkable features of wave propagation in deep water.

If we investigate the origin of this acceleration, we see that it is related to the fact that the phase velocity of water waves, $v_p = \sqrt{g/k}$ is different than the group velocity $v_g = (1/2)v_p$. A particle moving at the phase velocity is continually carried forward through the wave packet ($v_p > v_g$), into a region with a higher phase velocity. Note that this acceleration process requires a wave packet with a broad spectrum of wave numbers. For a narrow wave number spectrum, centered on ω_o and k_o, the usual linear expansion of the dispersion relation gives a wave packet of the form

$$f(x,t) = A(x - v_g t)\exp\left[i(k_o x - \omega_o t)\right] \tag{4}$$

for which the phase velocity $v_p = \omega_o/k_o$ stays constant.

The same dispersive acceleration process described above also occurs for whistler mode wave packets, such as the hook-shaped chorus emission in Figure 1. This acceleration process was first pointed out in the case of whistlers by Brice [1960]. For nearly parallel propagation it is easily verified [Helliwell, 1965] that the phase velocity of the whistler mode at low frequencies ($\omega \ll \omega_g$) is given by

$$v_p = c\,\frac{\sqrt{\omega_g \omega}}{\omega_p} \tag{5}$$

where ω_g is the electron gyrofrequency. It can also be shown that the group velocity is twice the phase velocity $v_g = 2v_p$. Thus, in contrast to water waves, where the planes of constant phase advance toward the front of the packet, for low frequency whistler mode waves the planes of constant phase recede toward the rear of the packet, opposite to the direction of motion of the packet. The effect on the velocity of a particle trapped in a chorus wave packet is illustrated in Figure 3, which shows the emission frequency as a function of the position s along a magnetic field line for two times t_1 and t_2. Because $v_p < v_g$ a particle initially trapped at A, near the point of minimum frequency (time t_1), moves toward the rear of the packet, so that later (time t_2) this particle is located at a point where the frequency has increased ($\omega_2 > \omega_1$). Since the phase velocity increases with increasing frequency, the velocity of the trapped particle increases in going from t_1 to t_2. By following the motion of a representative particle in various parts of the packet it can be seen that the acceleration is positive whenever $\partial\omega/\partial t > 0$ and negative (deceleration) whenever $\partial\omega/\partial t < 0$, where $\omega(t)$ is the instantaneous frequency at a fixed position.

From considerations of the relative motion of a particle trapped in a wave packet with instantaneous frequency $\omega(t)$, it is relatively straightforward to show that the general formula for the acceleration, valid for an arbitrary dispersion relation is

$$a = \frac{1}{k}\left(1 - \frac{v_p}{v_g}\right)^2 \frac{\partial\omega}{\partial t} \tag{6}$$

where v_p and v_g are the phase and group velocities at frequency ω. This formula correctly gives the acceleration of water waves and agrees with the equation given by Brice [1960] for the special case of whistler mode waves. Note from Equation 6 that a positive acceleration occurs whenever $\partial\omega/\partial t$ is positive, independent of the relationship between v_p and v_g, provided only that v_p and v_g not be the same. Thus, for the whistler mode a and $\partial\omega/\partial t$ have the same sign even when ω is not small compared to ω_g. The only difficulty for the whistler mode arises when $\omega = \omega_g/2$, where it can be shown that $v_p = v_g$. Below $\omega_g/2$, $v_g > v_p$, and above $\omega_g/2$, $v_g < v_p$. Normally, the chorus and hook emissions have frequencies well below $\omega_g/2$ so we are always in the regime where $v_g > v_p$.

The dependence of the acceleration on the sign of $\partial\omega/\partial t$ in Equation 6 is believed to explain one puzzling feature of the electrostatic bursts. Often, when a clearly defined hook-shaped chorus emission occurs, as in Figure 1, the electrostatic emission

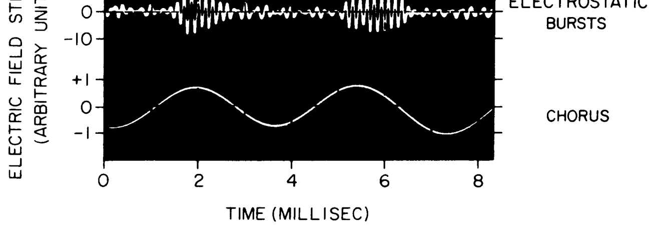

Fig. 2. Simultaneous waveforms showing that the chorus wave modulates the intensity of the electrostatic emissions. The electrostatic bursts are believed to be caused by electrons bunched in Landau resonance with the chorus wave.

only occurs on the rising portion of the hook, where $\partial\omega/\partial t$ is positive. According to the above analysis electrons trapped in this portion of the wave packet are being accelerated to higher velocities because $\partial\omega/\partial t$ is positive. This increase in the velocity of the trapped particles would be expected to enhance the electrostatic two-stream instability because it moves the particles upward in velocity, away from the main part of the thermal distribution which causes Landau damping.

Trapping and Escape Mechanisms

We now consider mechanisms by which electrons can be trapped and can escape from the potential well of a whistler mode wave packet. Computer simulations, such as those presented by Reinleitner et al. [1983], show that the most effective way of trapping a particle by a Landau resonance interaction is for the $E_\parallel$ field of the wave to be increasing with increasing time. This result is not unexpected since if the depth of the potential well is suddenly increased, a particle of the proper phase will be trapped if the differential velocity with respect to the wave is less than approximately $(eE_\parallel \lambda_\parallel /m)^{1/2}$, where $\lambda_\parallel$ is the wavelength parallel to the magnetic field. It is also important that the differential velocity not change too rapidly compared to the period of the wave. This latter condition favors capture near the equator, where the spatial gradients are the smallest.

Several mechanisms can cause the $E_\parallel$ field to increase. The first and most obvious mechanism is the temporal growth of the whistler mode wave packet. This growth, which is driven by the cyclotron loss-cone instability, occurs mainly near the equatorial plane where the particles can stay in cyclotron resonance the longest. The equatorial plane is therefore the favored region for trapping due to the temporal growth of the whistler mode wave. Second, as the wave propagates away from the equator the wave normal angle tends to increase [Burton and Holzer, 1974], thereby increasing $E_\parallel$. The increasing wave normal angle is expected to extend the trapping region somewhat beyond the main region of whistler mode growth near the equatorial plane. Eventually, Landau damping of the whistler mode wave should offset the effect of the increasing wave normal angle, at which point trapping would cease. However, a third mechanism can extend the trapping even farther from the equatorial plane. Because the group velocity of the wave packet is greater than the phase velocity, new particles are continually convected into the leading edge of the packet where they effectively see an increasing $E_\parallel$, thereby trapping some of these particles. This process is illustrated in Figure 4. The wave packet overtakes a particle moving at the phase velocity and traps this particle at A. The trapped particle is then carried through the packet, decelerating from A to B, where $\partial\omega/\partial t$ is negative, then accelerating from B to C, where $\partial\omega/\partial t$ is positive, and finally escaping at C. If the phase velocity at C is greater than at A, which is the case for most hook-shaped chorus emissions, the net effect is to accelerate the trapped particle to a higher energy. This process has many similarities to a laboratory linear accelerator. In fact, a somewhat similar process was once suggested by Sloan and Drummond [1973], for a laboratory accelerator.

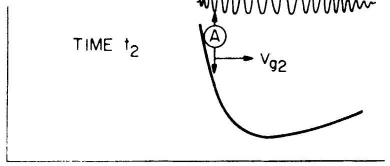

Fig. 3. The time evolution of a chorus wave packet showing that a particle in Landau resonance at point A moves toward the rear of the packet because the group velocity v_g is larger than the phase velocity v_p.

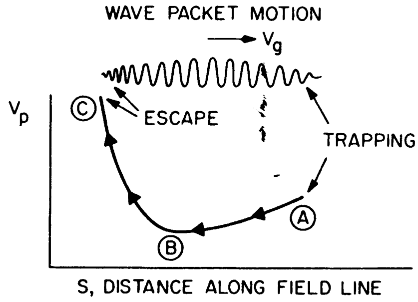

Fig. 4. The variation of the phase velocity of a trapped electron as it moves through a chorus wave packet from A to C. The particle is decelerated from A to B (where $\partial\omega/\partial t < 0$) and accelerated from B to C (where $\partial\omega/\partial t > 0$).

Numerous mechanisms exist that can cause trapped particles to escape from the wave packet. Because of the finite length of the wave packet and the fact that the group velocity is larger than the phase velocity all particles must eventually escape from the trailing edge of the wave packet, as indicated in Figure 4. The path length for this process to occur can be estimated by noting that for a 1 second chorus element the typical length of the wave packet (for an index of refraction of 20) is about 4.7 R_E. Since the phase velocity is approximately one-half the group velocity, all trapped particles are carried through the packet by the time the packet moves a distance of 9.4 R_E. Thus, for chorus in the outer regions of the magnetosphere (at $L = 10$, for example) all the electrons initially trapped at the equator will have escaped by the time the packet reaches the ionosphere. Because the magnetic field strength B increases as the wave approaches the Earth, conservation of the first adiabatic invariant shows that the trapped electrons gain energy as they move away from the equator. This energization process is illustrated in Figure 5 which shows the total energy of a trapped electron as a function of distance along the field line. The perpendicular energy, $W_\perp = \mu B$, where μ is the magnetic moment, increases directly with B until the particle escapes from the packet. Once the particle escapes, it mirrors, and then returns to the equator with more energy than it had when trapping occurred. This energization process occurs in addition to any acceleration that may occur because of changes due to inhomogeneous or dispersive effects. Note that a large change in the pitch angle can occur between trapping and escape. This pitch angle change is usually much larger than would occur for scattering by an incoherent wave of comparable amplitude.

The magnetic mirror force, $F = -\mu\partial B/\partial s$, can also cause trapped particles to escape from the wave packet. The escape condition is simply that the magnetic mirror force exceed the force due to the parallel electric field of the wave, $\mu\partial B/\partial s > eE_\parallel$. For a moderate equatorial pitch angle of $\alpha_0 \simeq 45°$, a resonant energy of $W \simeq 10^3$ eV, and an electric field of $E_\parallel = 100\ \mu V m^{-1}$, we estimate that an electron will escape via this mechanism at about $s = 4.3\ R_E$ along the $L = 10$ field line. Electrons with smaller equatorial pitch angles remain trapped farther from the equator. For $L = 10$, particles with equatorial pitch angles less than about 0.6° remain trapped all the way to the foot of the field line.

Other escape mechanisms also exist that are more difficult to evaluate. For example, because the trapping is inherently nonlinear, interactions between two or more waves can cause particles to escape. Also, small scale density irregularities could cause par-

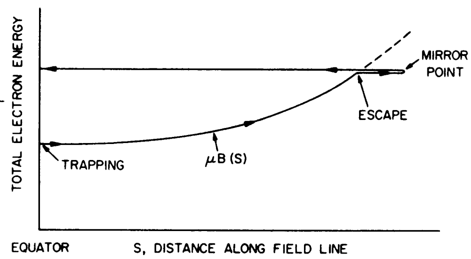

Fig. 5. The total energy of an electron trapped in Landau resonance at a constant velocity. The perpendicular energy increases in direct proportion to the magnetic field strength because of conservation of the first adiabatic invariant, $W_{\perp} = \mu B$.

ticles to escape, as could reflection of the wave at frequencies below the lower hybrid resonance. At the present level of analysis we have not attempted a detailed analysis of any of these escape mechanisms.

Discussion

We have discussed the trapping, acceleration and escape of electrons in Landau resonance with whistler mode wave packets in the outer regions of the Earth's magnetosphere. Both the theory and data suggest that electrons are being trapped and energized by this process. Landau resonance interactions with whistler mode wave packets are in many respects comparable to a linear accelerator, with particles being trapped, accelerated by changes in phase velocity, and ejected at higher energies. Changes in the magnetic field strength while a particle is trapped can also cause energization because of conservation of the first adiabatic invariant. The processes involved are remarkable in that they absorb energy and momentum from one class of particles via the cyclotron resonance, and transfer this energy and momentum to another class of particles via the Landau resonance. At present, it is not known to what extent this process may be responsible for the loss of energetic trapped electrons from the Earth's magnetosphere. Whistler mode chorus emissions and the associated electrostatic bursts are quite common and occur over large regions of the Earth's outer magnetosphere. Essentially identical electrostatic emissions are also observed in the magnetospheres of Jupiter and Saturn. Therefore, there is a good possibility that Landau resonance interactions are of considerable general importance. Similar Landau resonance interactions should also be produced by whistler mode signals from ground VLF transmitters.

Acknowledgements. The authors thank Professor R. A. Helliwell for pointing out the previous work on the dispersive acceleration mechanism by N. Brice. This research was supported by NASA through Grant NGL-16-001-043 from NASA Headquarters and Contract NAS5-26819 with Goddard Space Flight Center, and by the Office of Naval Research.

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(Received March 24, 1983;
accepted May 18, 1983.)