Theory of the Injun 5 Very-Low-Frequency Poynting Flux Measurements

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We present the theory of the VLF Poynting flux measurement technique used on the Injun 5 satellite. This technique consists of using one electric and one magnetic antenna, both oriented perpendicular to the geomagnetic field and to each other, to determine the direction of the VLF Poynting flux up or down the geomagnetic field. The conditions for which the Poynting flux direction determinations are valid are considered, including the effects of errors in the magnetic orientation of the spacecraft and the simultaneous presence of many waves.

This paper presents the theoretical basis for the VLF Poynting flux measurement technique used on the Injun 5 satellite. A preliminary discussion of the measurement theory and technique was given by *Gurnett et al.* [1969], and initial results of the measurements were presented by *Mosier and Gurnett* [1969]. This paper presents a more detailed theory of the measurement method, including an analysis of effects due to errors in the magnetic orientation of the spacecraft and an extension of the theory to include the simultaneous presence of many waves.

Poynting flux measurements of VLF radio noises in the magnetosphere are fundamentally important in establishing the source regions and emission mechanisms of these noises. The Poynting flux measurement method considered here consists of using one electric and one magnetic antenna, both oriented perpendicular to the geomagnetic field and to each other, to determine the direction of the Poynting flux, up or down the geomagnetic field. This Poynting flux measurement technique, which can be readily instrumented on a magnetically stabilized satellite, has the desirable feature of providing the basic Poynting flux information needed with the minimum number of antennas and signal processing. A comparable determination with a nonmagnetically oriented antenna system would require a minimum of four antennas (see discussions by *Storey* [1967]; *Shawhan* [1970]) and considerably more costly signal processing on the ground.

The Poynting flux measurement method described here was successfully used on the low-altitude(677 to 2528 km) polar-orbiting Injun 5 satellite. A detailed description of the Injun 5 VLF experiment is given by *Gurnett et al.* [1969].

POYNTING FLUX MEASUREMENTS FOR A SINGLE PLANE WAVE

The theoretical basis for the Injun 5 type of Poynting flux measurement will be derived for a single plane wave. The restriction to a single plane wave limits our results to whistlers, discrete VLF emissions, and other phenomena for which it is reasonably certain from the temporal characteristics that only a single wave is present. An extension of these results to situations where many waves propagating in different directions are present simultaneously, such as may occur for VLF hiss and other steady state noise phenomena, is presented in the next section.

The xyz coordinate system and notation used to describe the wave corresponds to that used by *Stix* [1962], in which the z axis is parallel to the static magnetic field and the wave normal is in the x–z plane at an angle θ relative to the static magnetic field. The electric and magnetic antenna axes are assumed to be oriented parallel to the x' and y' axes, respectively, of the spacecraft $x'y'z'$ coordinate system. When the spacecraft is magnetically oriented, the z and z'

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axes coincide. Since the wave vector can be at any arbitrary azimuth angle around the static magnetic field, the spacecraft coordinate system must be rotated at an arbitrary angle ψ relative to the wave coordinate system as shown in Figure 1. Any deviation from magnetic alignment leads to a displacement of the electric and magnetic antenna axes from the x–y plane by angles δ_E and δ_B, respectively, as shown in Figure 1.

By using the homogeneous equation for the electric field vector given by *Stix* [1962] and Maxwell's second relation, the fields in the wave coordinate system are given by (in mks units)

$$\begin{aligned} E_x &= e_x \cos(-\omega t) & B_x &= b_x \sin(-\omega t) \\ E_y &= e_y \sin(-\omega t) & B_y &= b_y \cos(-\omega t) \\ E_z &= e_z \cos(-\omega t) & B_z &= b_z \sin(-\omega t) \end{aligned} \quad (1)$$

where

$$e_x = E_0$$

$$e_y = \left(\frac{D}{S - n^2}\right)E_0$$

$$e_z = -\left(\frac{n^2 \cos\theta \sin\theta}{P - n^2 \sin^2\theta}\right)E_0$$

$$b_x = \frac{-n\cos\theta}{c}\left(\frac{D}{S - n^2}\right)E_0 \quad (2)$$

$$b_y = \frac{n\cos\theta}{c}\left(\frac{P}{P - n^2\sin^2\theta}\right)E_0$$

$$b_z = \frac{n\sin\theta}{c}\left(\frac{D}{S - n^2}\right)E_0$$

and S, P, and D are the dielectric tensor elements defined by *Stix* [1962].

The instantaneous Poynting flux along the static magnetic field, which is the component of interest, is given by

$$S_z = E_x H_y - E_y H_x$$

$$= \frac{1}{\mu_0}(E_x B_y - E_y B_x) \quad (3)$$

By computing time averages, denoted by angle brackets, over one complete period of cos $(-\omega t)$, the average Poynting flux along the static magnetic field is given by

$$\langle S_z \rangle = \frac{1}{2} E_0^2 \frac{n\cos\theta}{\mu_0 c} \cdot \left[\frac{P}{P - n^2\sin^2\theta} + \left(\frac{D}{S - n^2}\right)^2\right] \quad (4)$$

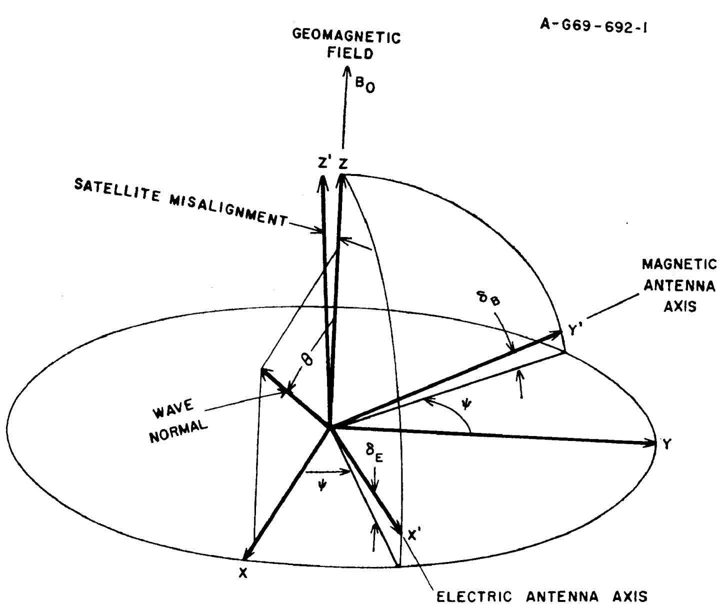

Fig. 1. Antenna and wave normal orientations showing the azimuthal rotation angle ψ and the antenna misalignment angles δ_E and δ_B.

The objective is to determine the sign of $\langle S_z \rangle$ in equation 4 from the measurements involving $E_{x'}$ and $H_{y'}$ in the spacecraft coordinate system. By using the appropriate coordinate transformation equations from the unprimed (wave) coordinates to the primed (spacecraft) coordinates, the time average of the cross product $E_{x'}H_{y'}$, which can be determined from the observed field, is given by

$$\langle E_{x'}H_{y'} \rangle = \frac{1}{2} E_0^{\,2} \frac{n \cos \theta}{\mu_0 c} \cdot \left[\frac{P}{P - n^2 \sin^2 \theta} \cos^2 \psi + \left(\frac{D}{S - n^2} \right)^2 \sin^2 \psi + \Delta \right] \qquad (5)$$

where

$$\Delta = -\left(\frac{n^2 \cos \theta \sin \theta}{P - n^2 \sin^2 \theta} \right) \left(\frac{P}{P - n^2 \sin^2 \theta} \right) \cdot \cos \psi \sin \delta_E + \left(\frac{D}{S - n^2} \right)^2 \sin \psi \tan \theta \sin \delta_B \qquad (6)$$

In obtaining equations 5 and 6, it has been assumed that the deviations from magnetic alignment are small so that $\cos \delta_E \simeq \cos \delta_B \simeq 1$. The term Δ represents the effect of any deviation from magnetic alignment. For exact magnetic alignment, $\delta_E = \delta_B = 0$ and $\Delta = 0$.

By comparing equations 4 and 5, we see that $\langle E_{x'}H_{y'} \rangle$ is proportional to $\langle S_z \rangle$,

$$\langle E_{x'}H_{y'} \rangle = G(\theta, \psi) \langle S_z \rangle \qquad (7)$$

where the proportionality factor $G(\theta, \psi)$ is given by

$$G(\theta, \psi) = \frac{\left(\dfrac{P}{P - n^2 \sin^2 \theta} \right) \cos^2 \psi + \left(\dfrac{D}{S - n^2} \right)^2 \sin^2 \psi + \Delta}{\left(\dfrac{P}{P - n^2 \sin^2 \theta} \right) + \left(\dfrac{D}{S - n^2} \right)^2} \qquad (8)$$

From equation 7 it is evident that the sign of the z component of the Poynting flux, $\langle S_z \rangle$, will be the same as the sign of $\langle E_{x'}H_{y'} \rangle$ if the proportionality factor $G(\theta, \psi)$ is positive for all wave normal angles. The conditions for which $G(\theta, \psi)$ is positive are considered below.

Exact magnetic alignment. We first consider the case of exact magnetic alignment, for which $\Delta = 0$. As is readily determined from equation 8, $G(\theta, \psi)$ is positive for all angles θ and ψ whenever $P < 0$. Since $P = 1 - f_{pe}^2/f^2$, this inequality is satisfied whenever the wave frequency, f, is less than the electron plasma frequency, f_{pe}. Since the electron plasma frequency is typically from 30 kHz to 1 MHz within the magnetosphere, this condition is always satisfied at VLF frequencies. Thus, for exact magnetic alignment the direction of the Poynting flux, up or down the geomagnetic field, can be determined directly from the sign of the correlation $\langle E_{x'}H_{y'} \rangle$.

It should be noted from equation 7 that only two field quantities, $E_{x'}$ and $H_{y'}$, are necessary to determine the Poynting flux direction (up or down the static magnetic field), whereas the z component of the Poynting flux as given by equation 3 involves four field quantities: E_x, E_y, H_x, and H_y. The reduction in the number of field measurements required, while still allowing determination of the sign of the Poynting flux, has been made at the expense of the ability to determine the magnitude of the Poynting flux, since the proportionality factor $G(\theta, \psi)$ involves the wave normal angle θ and the azimuth angle ψ, which are unknown.

Deviations from exact magnetic alignment. When deviations from magnetic alignment occur, the term Δ in equation 8 is non-zero. Depending on the detailed misalignment angles, δ_E and δ_B, the sign of Δ can be either positive or negative. If Δ is negative and the misalignment is sufficiently large, then the sign of $G(\theta, \psi)$ may be negative for some wave normal directions, in which case the Poynting flux direction determination can be in error.

The condition that $G(\theta, \psi)$ be positive when misalignment errors are included can be written

$$\Delta > -F \qquad (9)$$

where F is the positive term (when $P < 0$)

$$F = \left(\frac{P}{P - n^2 \sin^2 \theta} \right) \cos^2 \psi + \left(\frac{D}{S - n^2} \right)^2 \sin^2 \psi \qquad (10)$$

in the numerator of equation 8.

The constraints imposed on the allowable magnetic misalignment are best illustrated by considering the limiting cases of $\psi = 0$ and $\psi = \pi/2$.

1. For $\psi = 0$ the condition $\Delta > -F$ simplifies to

$$\left(\frac{n^2 \sin \theta \cos \theta}{P - n^2 \sin^2 \theta}\right) \sin \delta_E < 1 \qquad (11)$$

At VLF frequencies in the ionosphere and magnetosphere the absolute value of P in the denominator of equation 11 is normally very large, typically 10^4 to 10^6. Thus, except for very large refractive indices the condition given by equation 11 is always satisfied, even for sizable misorientations of the electric antenna. The physical reason for this insensitivity to the electric antenna orientation is that when $|P| \gg n^2$ the large conductivity parallel to the static magnetic field constrains the electric field to a plane nearly perpendicular to the static magnetic field (see equation 2 for the ratio e_z to e_x). Thus, misalignments of the electric antenna from the x–y plane do not significantly affect the phase of the detected electric field signal, except for misalignment angles so large as to violate the original assumption of small misalignment angles.

For very large refractive indices ($n^2 > |P|$), equation 11 can be violated only for small wave normal angles ($\theta < \delta_E$). However, a large refractive index at a small wave normal angle can only occur near the electron gyrofrequency for the whistler mode or near the ion cyclotron frequency for the ion cyclotron mode. It is expected that this condition could be identified from the large dispersion, the large magnetic to electric field intensity, and other effects that are observed near a gyrofrequency resonance.

2. For $\psi = \pi/2$ the condition $\Delta < -F$ simplifies to

$$\tan \theta \sin \delta_B < 1 \qquad (12)$$

For small misorientations of the magnetic antenna, equation 12 is satisfied for all wave normal angles θ less than $(\pi/2 - \delta_B)$. Since δ_B is typically of the order of 10° or less for Injun 5, errors in the Poynting flux determination due to misalignment of the magnetic antenna on this spacecraft can occur only for waves propagating very nearly perpendicular to the static magnetic field. Physically this error occurs when the magnetic antenna axis becomes perpendicular to the plane of rotation of the wave magnetic field.

The form of equation 6 for Δ provides a simple method of testing whether magnetic alignment errors are causing an error in the Poynting flux direction determination. As can be seen from equation 6, the sign of Δ is determined by the signs of δ_E and δ_B. Of the four possible combinations of signs for δ_E and δ_B, at least one will result in a positive sign for Δ, in which case $G(\theta, \psi)$ must be positive. Thus, if $\langle E_{x'} H_{y'} \rangle$ is consistently observed to have the same sign for all four combinations of the signs of δ_E and δ_B, then it can be concluded that the magnetic misalignment errors are not producing an error in the Poynting flux determination. For a magnetically oriented satellite the angles δ_E and δ_B oscillate about zero with a characteristic period on the order of a few minutes, so that all possible sign combinations for δ_E and δ_B can be easily obtained. The angles δ_E and δ_B can be determined from a magnetometer on the spacecraft.

EXTENSION OF THE THEORY TO MULTIPLE WAVES

When many waves with different wave normal angles are present simultaneously, the interpretation of the measured correlation $\langle E_{x'} H_{y'} \rangle$ in terms of the Poynting flux must be carefully considered, since the proportionality factor $G(\theta, \psi)$ is usually not isotropic, thereby giving a nonuniform weighting to different wave normal directions. As will be shown, if $\langle E_{x'} H_{y'} \rangle$ is observed to be positive (or negative), then we conclude that at least some of the waves must have a Poynting flux in the positive (or negative) z direction. There may also be waves with Poynting fluxes in the opposite direction, however, and the relative intensity of these waves cannot be determined without further information on the wave normal angles involved. Only under certain restricted conditions is it possible to determine the sign of the average Poynting flux of all the waves. These conclusions, which were mentioned but not discussed by *Mosier and Gurnett* [1969], are derived and discussed in detail below.

If several waves are present, then the measured correlation $\langle E_{x'} H_{y'} \rangle$ can be expanded as

$$\langle E_{x'} H_{y'} \rangle = \langle \sum_i E_{x'i} \sum_i H_{y'i} \rangle \tag{13}$$

$$\langle E_{x'} H_{y'} \rangle = \sum_i \langle E_{x'i} H_{y'i} \rangle + \sum_{i \neq j} \langle E_{x'i} H_{y'j} \rangle$$

where the summations are over the individual waves. If it is assumed that the individual waves are uncorrelated, then the second summation in equation 13 will average to zero yielding

$$\langle E_{x'} H_{y'} \rangle = \sum \langle E_{x'i} H_{y'i} \rangle \tag{14}$$

The assumption of uncorrelated waves is generally considered to be reasonable, because waves with different wave normal directions can be assumed to have originated from different independent source regions. Possible violations of this assumption can occur if there are multiple ray paths from the source to the satellite owing to reflections or other refraction effects within the ionosphere.

For the individual waves we have from equation 7

$$\langle E_{x'i} H_{y'i} \rangle = G(\theta_i, \psi_i) \langle S_{zi} \rangle \tag{15}$$

which when combined with equation 14 gives

$$\langle E_{x'} H_{y'} \rangle = \sum G(\theta_i, \psi_i) \langle S_{zi} \rangle \tag{16}$$

Equation 16 shows that the contribution to $\langle E_{x'} H_{y'} \rangle$ from each wave is weighted by the function $G(\theta, \psi)$. The proportionality factor $G(\theta, \psi)$ therefore plays the role of an effective 'antenna pattern' for the measurement of the z component of the Poynting flux. Since $G(\theta, \psi)$ is positive (subject to the restrictions discussed in the previous section), it is evident that if $\langle E_{x'} H_{y'} \rangle$ is observed to be positive (or negative), then it can be concluded that at least some of the waves must have a Poynting flux in the positive (or negative) z direction. Only if $G(\theta, \psi)$ is isotropic, however, will the measured correlation be proportional to the sum of the Poynting flux of the individual waves.

The weighting function $G(\theta, \psi)$ has been investigated to determine when it is possible to interpret the sign of $\langle E_{x'} H_{y'} \rangle$ as the sign of the average Poynting flux. To simplify the discussion we assume that the spacecraft is perfectly aligned ($\Delta = 0$) and that the condition $|P| \gg n^2 \sin^2 \theta$ (with $P < 0$) is satisfied. From the algebraic form of the equation for $n(\theta)$ [see *Stix*, 1962] it can be shown that $G(\theta, \psi)$ is symmetric about the plane $\theta = 90°$.

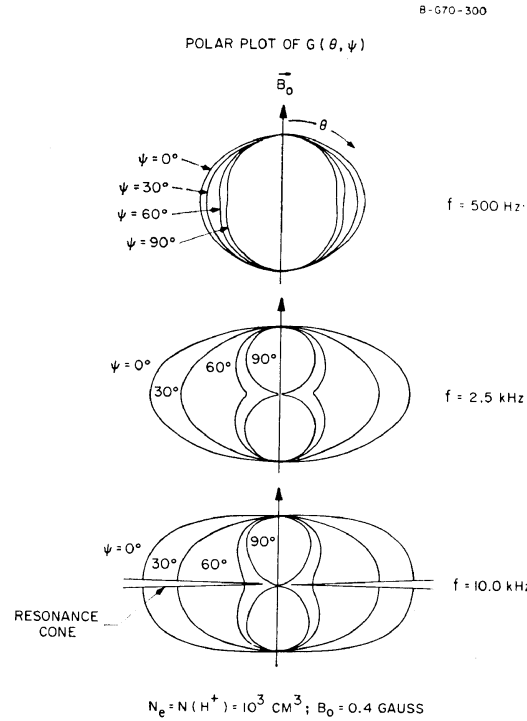

Fig. 2. Polar plots of the proportionality factor $G(\theta, \psi)$.

Therefore, upgoing and downgoing waves with the same wave normal angle are weighted equally. The function $G(\theta, \psi)$ is also symmetric about the $\psi = 0°$ and $\psi = 90°$ planes. At $\psi = 90°$, the function $G(\theta, \psi)$ approaches zero as the refractive index approaches infinity at the resonance cone angle. If there is no resonance cone, then $G(\theta, \psi)$ is non-zero and positive for all wave directions. It can be shown that if $G(\theta, \psi)$ is averaged over ψ, weighting all values of ψ equally, the resulting average is independent of the wave normal angle. Near any gyrofrequency, where D and S approach infinity, $G(\theta, \psi)$ becomes isotropic for the fast mode of propagation.

Polar plots computed to illustrate the general features of $G(\theta, \psi)$ are shown in Figure 2 at frequencies of 500 Hz and 2.5 and 10.0 kHz for typical ionospheric plasma parameters. At 500 Hz, which is close to the proton gyrofrequency at 607 Hz, $G(\theta, \psi)$ is seen to be nearly isotropic, as expected from the above discussion. At higher frequencies the weighting function becomes more anisotropic (doughnut shaped), with a sharp minimum at $\theta = 90°$ and $\psi = 90°$

or 270°. Above the lower hybrid resonance frequency, which is at 6.2 kHz in this case, $G(\theta, \psi)$ goes to zero at the resonance cone angle for $\psi = 90°$ or 270°.

From these results the following conclusions can be made concerning the possibility of determining the sign of the average z component of the Poynting flux from the sign of $\langle E_{x'}H_{y'}\rangle$.

1. Near any gyrofrequency $G(\theta, \psi)$ becomes nearly isotropic for the fast mode of propagation. Therefore, near the proton gyrofrequency, for example, the sign of the average Poynting flux, up or down the magnetic field, can be determined for the extraordinary (whistler) mode.
2. Because $G(\theta, \psi)$ averaged over all azimuth angles is independent of the wave normal angle, the sign of the average Poynting flux, up or down the static magnetic field, can be determined if there is reason to believe that the wave normal angles are uniformly distributed in the angle ψ. For example, if for a certain phenomenon $\langle E_{x'}H_{y'}\rangle$ is always observed to be positive (or negative) for successive observations in which the satellite has rotated randomly in azimuth about the static magnetic field (to assure a uniform distribution to ψ values), it may be concluded that the average value of the z component of the Poynting flux is positive (or negative).

Discussion

Although this Poynting flux measurement technique is ideally suited for magnetically stabilized satellites, the same technique and theory can also be applied to various other geometries. Since the electric antenna orientation relative to the static magnetic field is not as critical as the magnetic antenna orientation, it may be adequate to only align the magnetic antenna perpendicular to the geomagnetic field. For example, on an earth stabilized spacecraft in a low earth orbit the magnetic antenna axis could be oriented east–west (which would always be nearly perpendicular to the static magnetic field), and the electric antenna could be oriented north–south. This orientation would be expected to give good Poynting flux data at middle and high latitudes where the geomagnetic field is nearly vertical. On a spinning satellite the magnetic antenna axis could be oriented perpendicular to the spin axis, so that the magnetic antenna axis will be perpendicular to the static magnetic field twice per rotation. At these times Poynting flux measurements can be made, if the electric antenna does not make too large an angle with the static magnetic field.

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