

An Experimental Study of Very-Low-Frequency Mode Coupling and Polarization Reversal

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Below the proton gyrofrequency, both polarization reversal and mode coupling of the right- and left-hand modes of propagation can occur. In this paper an experimental study of polarization reversal and mode coupling of electron and proton whistlers is presented. The occurrence of polarization reversal for a whistler signal observed in the ionosphere is indicated by the presence of a proton whistler. Mode coupling between the right- and left-hand modes of propagation is indicated by the occurrence of both electron and proton whistler signals at the same frequency. Mode coupling is observed to occur most frequently over a range of about 35°–55° magnetic latitude. Below about 35° magnetic latitude, polarization reversal is the predominant effect, whereas above about 55° magnetic latitude neither mode coupling nor polarization reversal occur and proton whistlers are not observed. These results are compared with existing theories to explain this latitude dependence.

Polarization reversal and mode coupling of electromagnetic waves in the ionosphere has been discussed by Gurnett *et al.* [1965] and Jones [1968, 1969] in connection with ion cyclotron whistlers observed by satellites [Smith *et al.*, 1964]. Gurnett and his co-workers considered a collisionless model of the ionosphere in explaining the polarization reversal of ion cyclotron whistlers and showed that mode coupling must be included to explain the observed transmission of both electron and proton whistler signals from the ground to the satellite at frequencies below the proton gyrofrequency. They describe mode coupling in terms of a critical angle θ_c , where θ is the angle between the geomagnetic field \mathbf{B}_0 and the wave-normal direction \mathbf{K} . An upgoing electron whistler propagating near the cone of angles defined by θ_c results in both electron and ion whistlers being present above the 'cross-over' altitude, the altitude at which the refractive indices of the two modes are equal. For the model used by Gurnett *et al.*, θ_c is of the order of 10°.

Jones [1968, 1969] has further extended the work of Gurnett *et al.* by showing in detail how the wave polarization varies near the crossover altitude, and how the wave-normal angle and the critical-coupling angle control polarization reversal and mode coupling phenomena. Using

the ion density profile of Gurnett *et al.* and a realistic model of collision frequencies versus altitude, Jones has computed values of θ_c at the crossover altitude as a function of frequency and latitude. His calculations predicted a marked dependence of mode coupling and polarization reversal on latitude. It is the purpose of this paper to report on an experimental investigation of mode coupling and polarization reversal of electron and proton whistlers and to compare these results with Jones' predictions.

POLARIZATION REVERSAL AND MODE COUPLING

Mode coupling of ion cyclotron whistlers in the ionosphere can be described using Forsterling's coupled equations for vertical incidence in an inhomogeneous, anisotropic medium [Budden, 1961]:

$$\begin{aligned} F_0'' + F_0(n_0^2 + \psi^2) &= \psi'F_x + 2\psi F_x' \\ F_x'' + F_x(n_x^2 + \psi^2) &= \psi'F_0 + 2\psi F_0' \end{aligned} \quad (1)$$

where F_0 and F_x are proportional to the fields in the ordinary and extraordinary modes, respectively. The prime denotes $1/K \partial/\partial z$, where $K = n\omega/c$ is the wave number and z is the vertical coordinate. The indices of refraction for the respective modes are n_0 and n_x , and ψ is the coupling parameter, so called because when $\psi = 0$ the equations are independent and the two modes propagate independently,

while if $\psi \neq 0$, the equations are coupled and there is interaction between the modes. ψ has the form

$$\psi = \frac{\rho_0'}{(\rho_0^2 - 1)} = \frac{\rho_r'}{(\rho_r^2 - 1)} \quad (2)$$

where ρ is the polarization, E_y/E_x , defined in terms of the components of the electric field that lie in the wavefront.

When consideration is given as to how the coupling parameter ψ can become large, two possibilities occur: first, the denominator of equation 2 can become small (with the numerator non-zero), or second, the numerator of equation 2 can become large if the polarization changes sufficiently rapidly with height. The first condition, for which ψ approaches infinity, is called the critical-coupling condition and occurs when $\rho = \pm 1$ with ρ' non-zero. This condition has been extensively investigated by Gurnett *et al.* [1965] and Jones [1968]. The second condition, for which ψ is large because of the rapid change in polarization with height, requires the use of a computer for further investigation. Full wave solutions for this type of coupling have been obtained by Wang [1970] and are discussed in an accompanying paper.

When collisions are neglected, the polarization of a propagating wave is purely imaginary and the critical-coupling condition cannot be satisfied. However, when collisions are considered, the critical-coupling condition can be satisfied provided (1) the wave frequency is equal to the crossover frequency, and (2) the wave-normal angle is equal to the critical-coupling angle. The crossover frequency corresponds to the condition $D = 0$ in the notation of Stix [1962]. Smith and Brice [1964] have shown that at the crossover frequency the polarizations of the two modes are interchanged. This polarization reversal process explains how left-hand polarized proton whistlers can be generated from right-hand polarized short fractional hop whistlers. A typical variation of the crossover frequency ($D = 0$), the proton gyrofrequency ($L = \infty$), and various other characteristic frequencies with altitude for a mid-latitude model ionosphere is shown in Figure 1. When collisions are considered, Jones [1968] has shown that polarization reversal may or may not occur at the crossover altitude depending on the wave-normal angle. If the wave-normal angle is less

than an angle called the critical-coupling angle, θ_c , then polarization reversal does not occur at the crossover altitude. The significance of the critical-coupling angle can be illustrated by plotting the complex polarization, ρ , as a function of altitude near the crossover altitude. Figure 2 is taken from Jones [1968] and shows such a plot for various wave normal angles using the ionospheric model of Figure 1 and a wave frequency of 400 Hz. The upper half of the complex ρ plane in Figure 2 is for the left-hand polarized mode and the lower half plane is for the right-hand polarized mode. (Jones uses the definitions $\rho = -i$ for right and $\rho = +i$ for left-hand polarization.) The critical-coupling angle for the conditions at the crossover altitude (772 km) is $\theta_c = 8.9^\circ$. From Figure 2 it is seen that for wave-normal angles less than θ_c (for example $\theta = 8^\circ$, labeled *A* in Figure 2) the polarization paths do not cross the real ρ axis, therefore, the sense of polarization, right- or left-hand, does not change as the wave passes the crossover altitude. For wave-normal angles greater than θ_c (for example $\theta = 10^\circ$, labeled *E* in Figure 2) the polarization path starts at $\rho = -i$ (right-hand) and terminates at $\rho = +i$ (left-hand), with the polarization reversal occurring as the wave passes the crossover altitude. In both cases illustrated ($\theta = 8^\circ$ and $\theta = 10^\circ$) the polarization does not come close to $\rho = \pm 1$ (the critical-coupling condition) so mode coupling is negligible.

For wave-normal angles close to $\theta_c = 8.9^\circ$, it can be seen from Figure 2 that the polarization path passes very close to $\rho = 1$ (inside the dotted circle, for example) and strong coupling will occur as the coupling parameter ψ becomes large at $\rho = +1$. For wave-normal angles close to the critical-coupling angle an upward traveling right-hand polarized wave below the crossover altitude will be split into right- and left-hand polarized waves upon passing the crossover altitude.

These conclusions of Gurnett *et al.* and Jones can be summarized as follows:

(a) $\theta > \theta_c$ indicates polarization reversal and negligible mode coupling. An upward traveling wave in the *R* mode changes smoothly into an upward traveling wave in the *L* mode upon passing the crossover altitude. If $|\theta - \theta_c|$ is small, weak coupling may generate a small

amount of energy in the R mode above the crossover altitude.

(b) $\theta \simeq \theta_c$ indicates critical coupling. An upward traveling wave in the R mode is split into two upward traveling waves in the R and L modes when ψ , the coupling parameter, becomes large as the polarization approaches $\rho = \pm 1$. A determination of the splitting of

the energy into the two modes requires a full-wave solution.

(c) $\theta < \theta_c$ indicates no polarization reversal and negligible mode coupling. An upward traveling wave in the R mode remains in the R mode. If $|\theta - \theta_c|$ is small, weak coupling may generate a small amount of energy in the L mode above the crossover altitude.

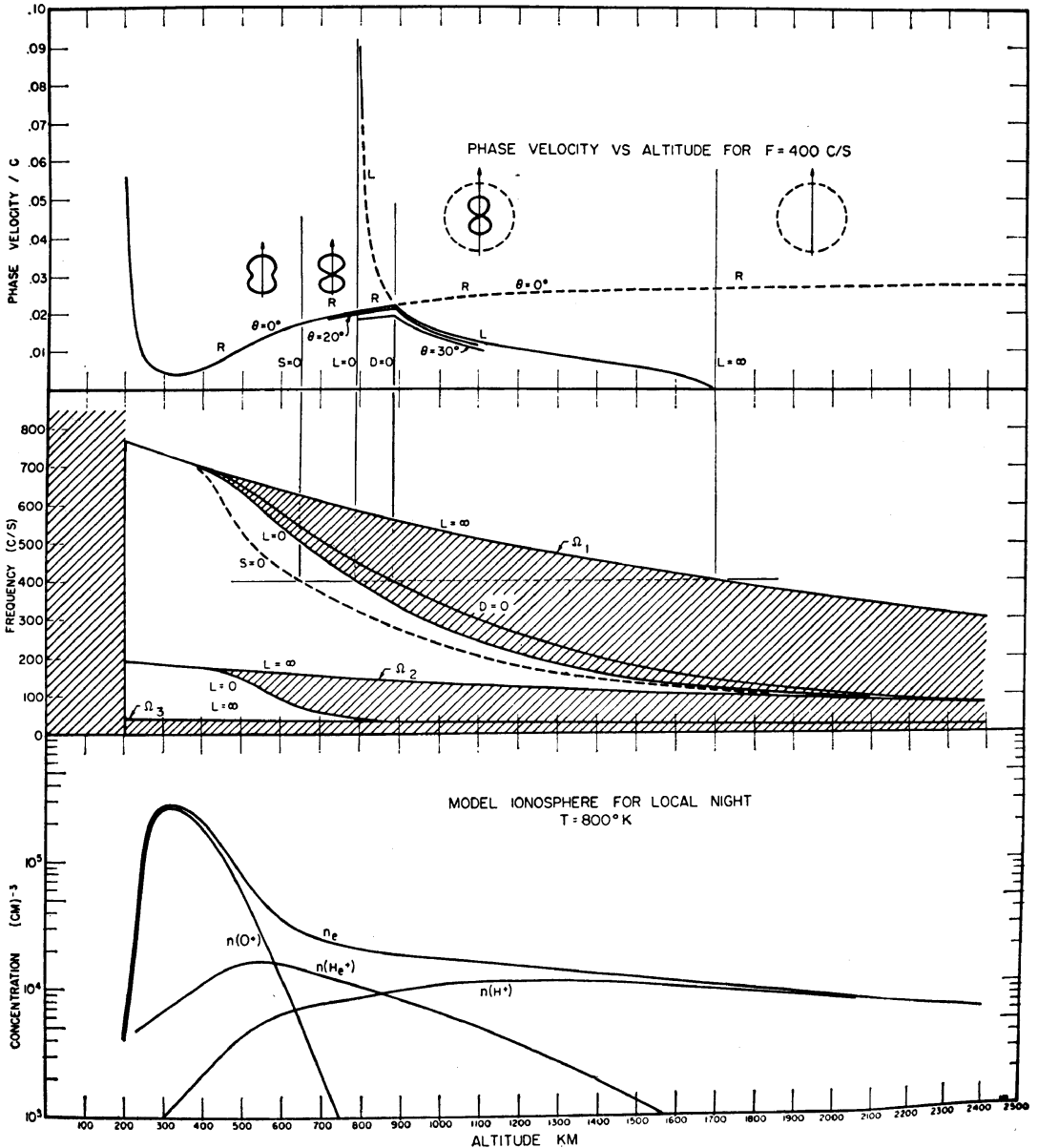


Fig. 1. Various critical frequencies versus altitude for a typical model ionosphere.

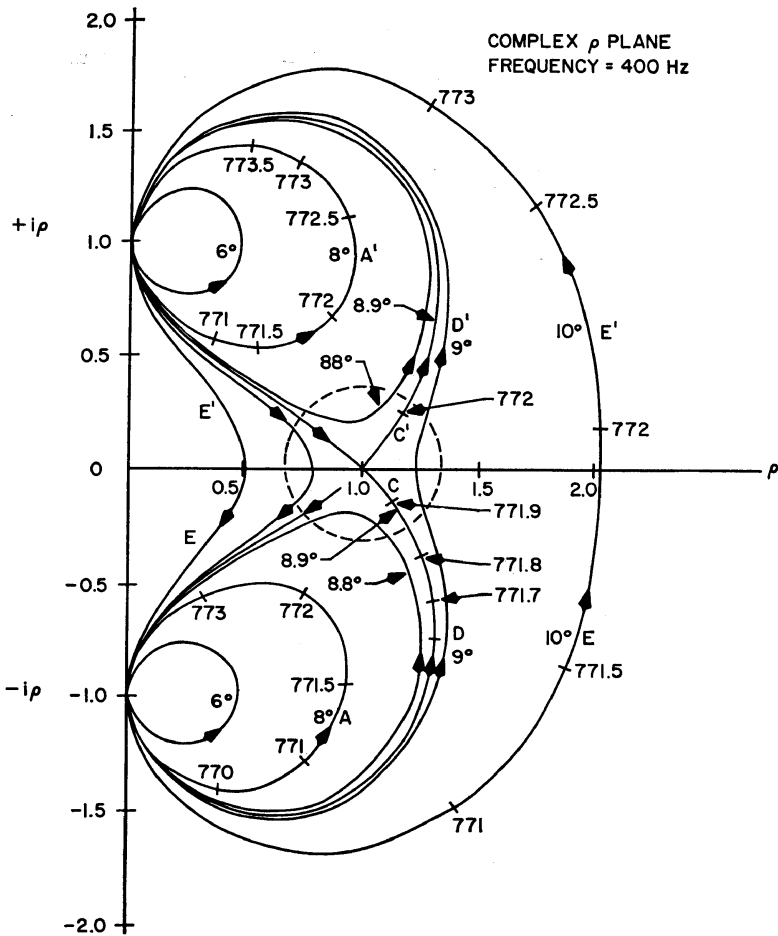


Fig. 2. Polarization path in the complex ρ plane versus altitude [Jones, 1968].

Because of the large refractive index in the ionosphere at VLF frequencies, the wave-normal direction of a whistler entering the ionosphere from below will be refracted to nearly vertical upon entering the ionosphere. In the absence of horizontal gradients, the direction between the wave normal of the whistler and the magnetic field direction is determined by the dip angle of the geomagnetic field. The polarization reversal and mode coupling processes discussed above are, therefore, expected to be strongly latitude dependent. Jones [1968, 1969] has considered the latitude dependence of polarization reversal and mode coupling for proton whistlers, and his results are summarized in Figure 3. Jones' classification considered three coupling types: fully formed proton whistlers, partially formed proton whistlers, and

no proton whistlers, corresponding respectively to wave-normal angles greater than, comparable to, and less than the critical-coupling angle. At low latitudes where the geomagnetic field makes a large angle relative to vertical, the wave-normal angle is greater than θ_c , so that only fully formed proton whistlers should be observed. At high latitudes where the geomagnetic field is nearly vertical, the wave-normal angle is less than θ_c , so that polarization reversal does not occur and no proton whistler should be observed. The computed transition latitudes between fully formed, partially formed, and no proton whistlers, are given by Jones in Figure 3. The transition latitudes are dependent on altitude because of the altitude dependence of the collision frequencies and other parameters which determine the critical-coupling angle.

CLASSIFICATION OF COUPLING TYPES

In the present study of mode coupling and polarization reversal, spectrograms of whistlers are classified according to the scheme given in Figure 4. These classifications have been selected after a detailed study of several hundred in-

dividual whistler spectrograms to insure that all types of observed mode coupling and polarization reversal phenomena can be included in this classification. In this respect, 'partially formed' proton whistlers, as defined by Jones and consisting of proton whistlers with cutoffs

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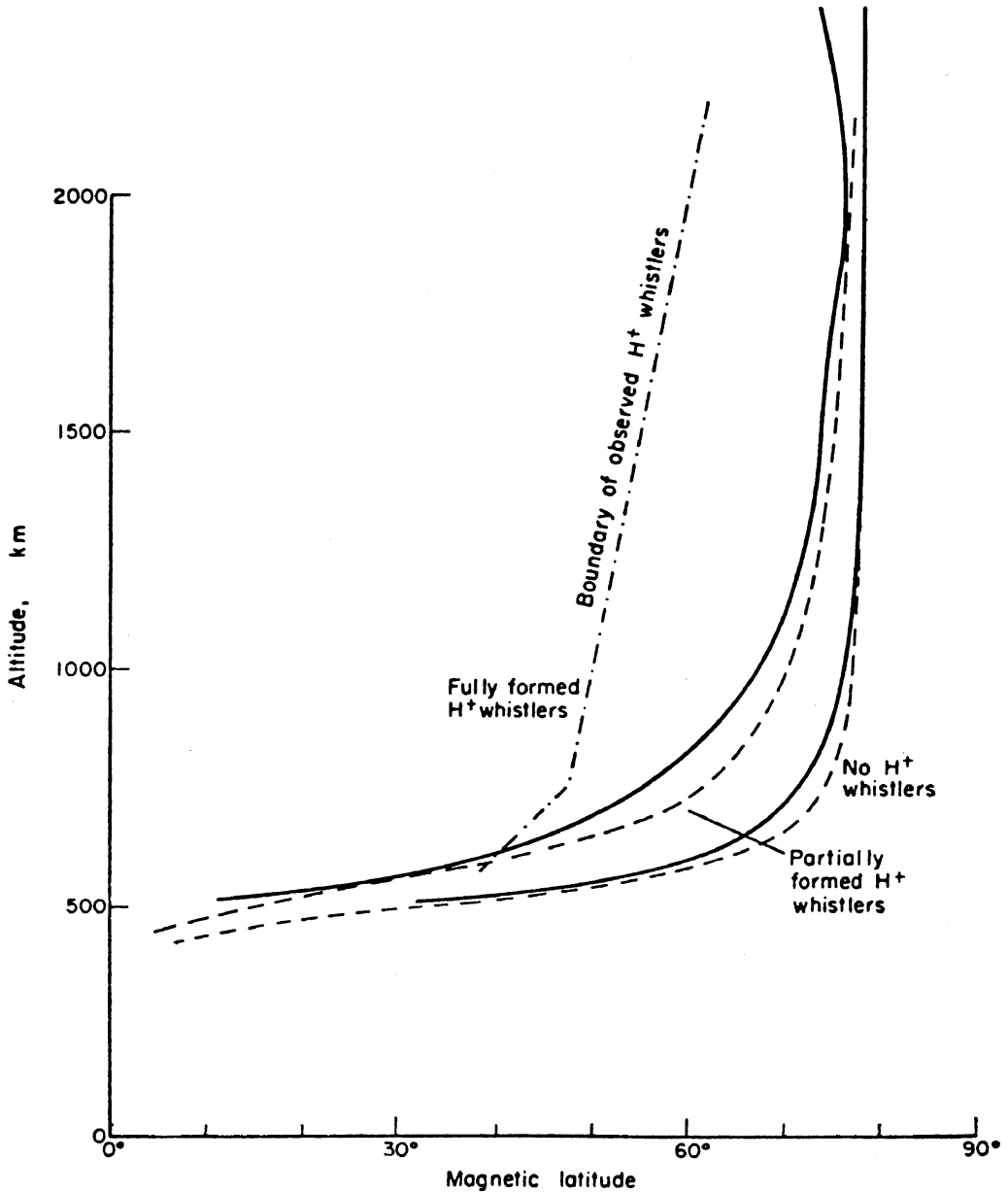


Fig. 3. Regions of fully-formed, partially-formed, and no-proton whistlers from Jones [1968].

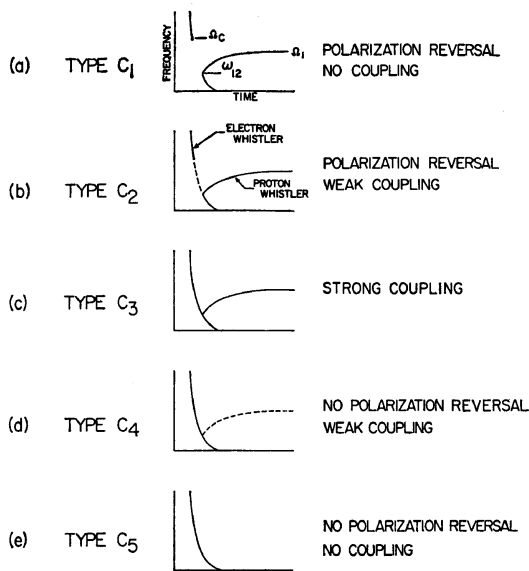


Fig. 4. Classification of coupling types from frequency-time spectrograms of proton whistlers.

at frequencies intermediate between the crossover frequency and the proton gyrofrequency, are rarely observed.

A. *Type C₁: Polarization reversal and no coupling.* In this coupling type, the observed spectrogram shows only a proton whistler between the crossover frequency, ω_{12} , and the proton gyrofrequency, Ω_1 . No electron whistler signal is seen in the frequency range from ω_{12} to Ω_c (see Figure 4a).

B. *Type C₂: Polarization reversal and weak coupling.* In this coupling type, a weak electron whistler is observed in the frequency range from ω_{12} to Ω_c (dashed line in Figure 4b), while most of the signal strength in the frequency range ω_{12} to Ω_1 is in the proton whistler. Thus, weak coupling and polarization reversal has occurred near the crossover altitude.

C. *Type C₃: Strong coupling.* In this coupling type, the signal strength of the proton and electron whistlers in the frequency range from ω_{12} to Ω_1 are observed to be approximately the same (see Figure 4c). Thus, strong coupling of the two waves has occurred at the crossover altitude.

D. *Type C₄: No polarization reversal and weak coupling.* In this coupling type, a weak proton whistler is observed in the frequency range from ω_{12} to Ω_1 , but most of the signal

strength remains in the electron whistler (see Figure 4d).

E. *Type C₅: No polarization reversal and no coupling.* In this coupling type, no proton whistler is observed above the crossover altitude. Neither polarization reversal nor mode coupling occurs. For this case, it is necessary that the electron whistler be observed at frequencies below the proton gyrofrequency so that the absence of a proton whistler can be ascertained. If the electron whistler is not observed at any frequency below the proton gyrofrequency, then the absence of a proton whistler could be caused by the absence of low-frequency components in the original lightning impulse. Cases for which no electron whistler signal was observed below the proton gyrofrequency were excluded from this study.

STATISTICAL SURVEY OF MODE COUPLING

Spectrograms of whistlers for this study were obtained from satellites Injun 3 and Injun 5, both of which are low altitude polar orbiting satellites. These satellites both carried very-low-frequency receivers covering the frequency range from a few tens of Hz to 10 kHz. Details of the VLF experiments on these satellites are given in Gurnett and O'Brien [1964] and Gurnett et al. [1969].

During a satellite pass where many whistlers are occurring, a progression of coupling types from C_1 at low geomagnetic latitudes, to C_5 at higher geomagnetic latitudes is generally observed. Figure 5 shows portions of such a pass near the transition latitudes. The magnetic antenna data shows that beginning at a geomagnetic latitude (GML) of 26.7° (The low latitude portion of this pass is not shown in Figure 5.) coupling type C_1 is being observed up to a latitude of about 39° . Between geomagnetic latitudes of 39° and 54° , coupling types C_2 , C_3 , and C_4 are primarily observed. Above 54° , mostly type C_5 is observed. On this pass, the transition latitudes defined by Jones and shown in Figure 3 are 39° and 54° , respectively. The 'smooth' behavior in coupling variation with latitude seen in this pass is not typical, however. Figure 6 shows a portion of a pass in which clear transition latitudes are not evident. Different coupling types are occurring close together and not in the expected latitudinal sequence. For example, near geomagnetic lati-

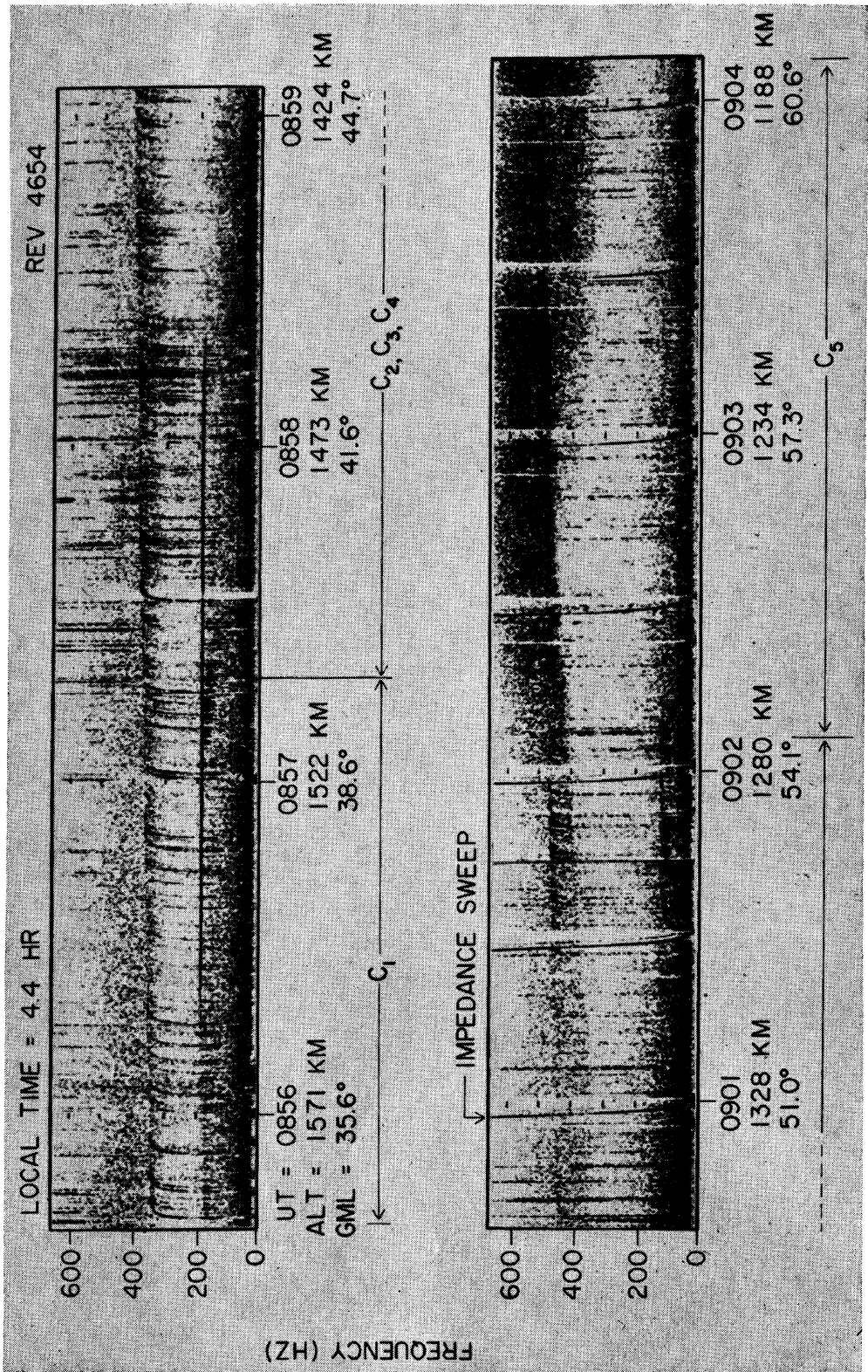


Fig. 5. Spectrograms of proton whistlers showing a clear transition of coupling types.

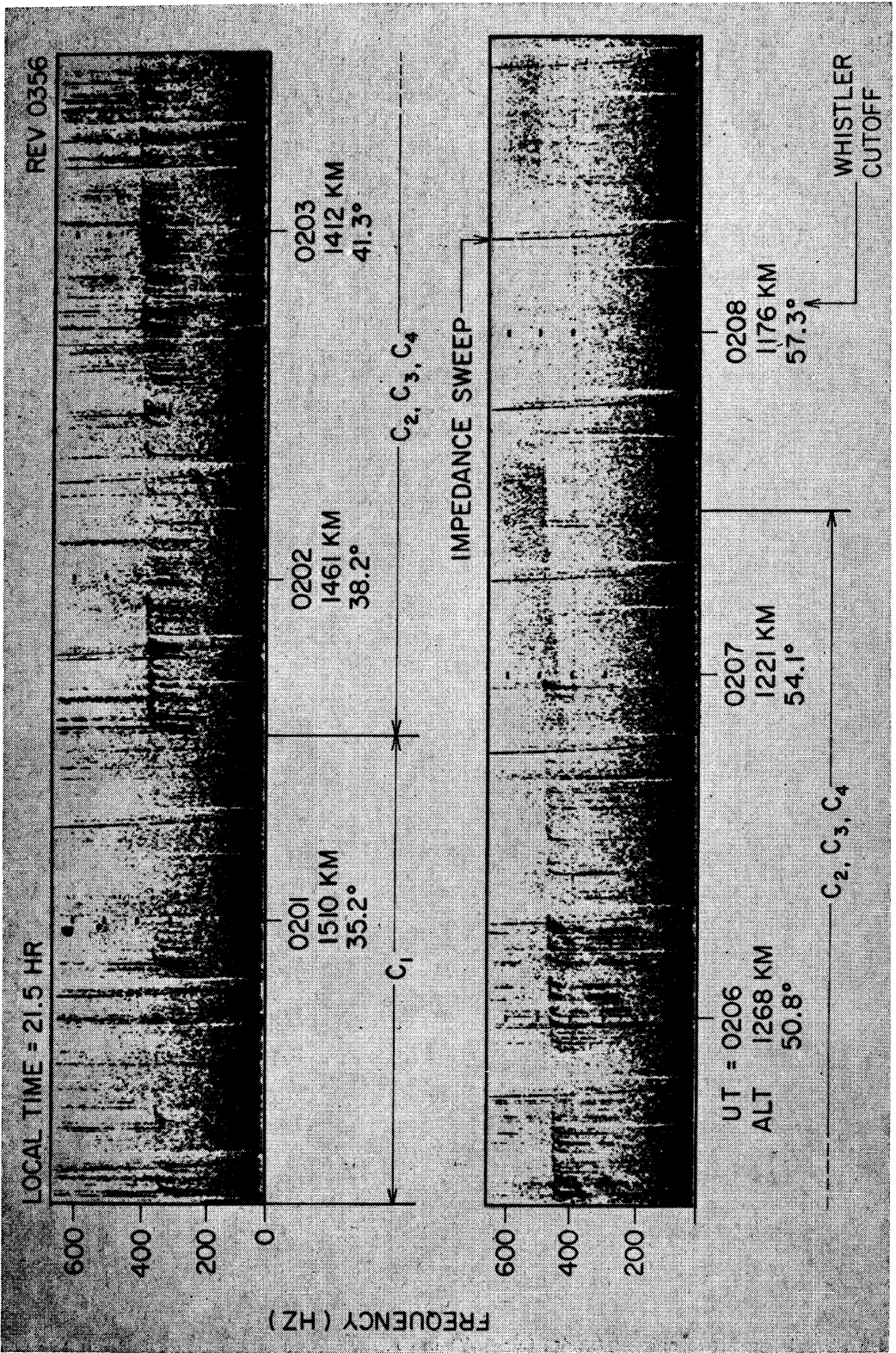


Fig. 6. Spectrograms of proton whistlers showing a mixture of coupling types with no clear transition of coupling types.

tude 50° , types C_1 are observed, while at about 40° , types C_2 and C_4 can be seen. The determination of transition latitudes must therefore be based on the frequencies of occurrence of coupling types C_2 , C_3 , and C_4 . This has been done for the transitions near 37° and 56° GML in Figure 6. Of about 60 summer night passes studied, there were only about 25% that showed distinct transition latitudes as in Figure 5. Most passes were of the type shown in Figure 6. Most passes also show a latitudinal cutoff of whistler signals in the vicinity of 60° geomagnetic latitude (indicated in Figure 6 at about 57° GML), which has been associated with the position of the plasmopause [Carpenter *et al.*, 1968; Taylor *et al.*, 1969; Heyborne *et al.*, 1969].

In Figure 7, 30 summer night passes have been plotted. The portion of each pass between the transition latitudes is shown by the solid line. The lines A and A' are the average boundaries marking out the transition region. The

portions of each pass shown by the dashed and dotted lines correspond to the occurrence of primarily C_1 and C_5 coupling types, respectively. A latitudinal cutoff in whistler activity is indicated by a cross on each pass in which it was observed to occur. The transition latitudes computed by Jones are also indicated in Figure 7 for comparison.

In addition to mode coupling, another important feature related to the critical coupling angle θ_c is the electron whistler cutoff frequency Ω_e , indicated in Figure 4. Jones [1969] shows that, when collisions are included, Ω_e is the frequency for which the critical coupling angle θ_c is equal to the wave-normal angle θ . Since θ_c depends on magnetic latitude and altitude, Ω_e also varies with magnetic latitude and altitude. In Figure 8, Jones' curve for Ω_e is reproduced as the solid line. Beside each point is written the altitude above which the corresponding value of Ω_e should be observed. Observed values of Ω_e are also plotted on Figure 8 for

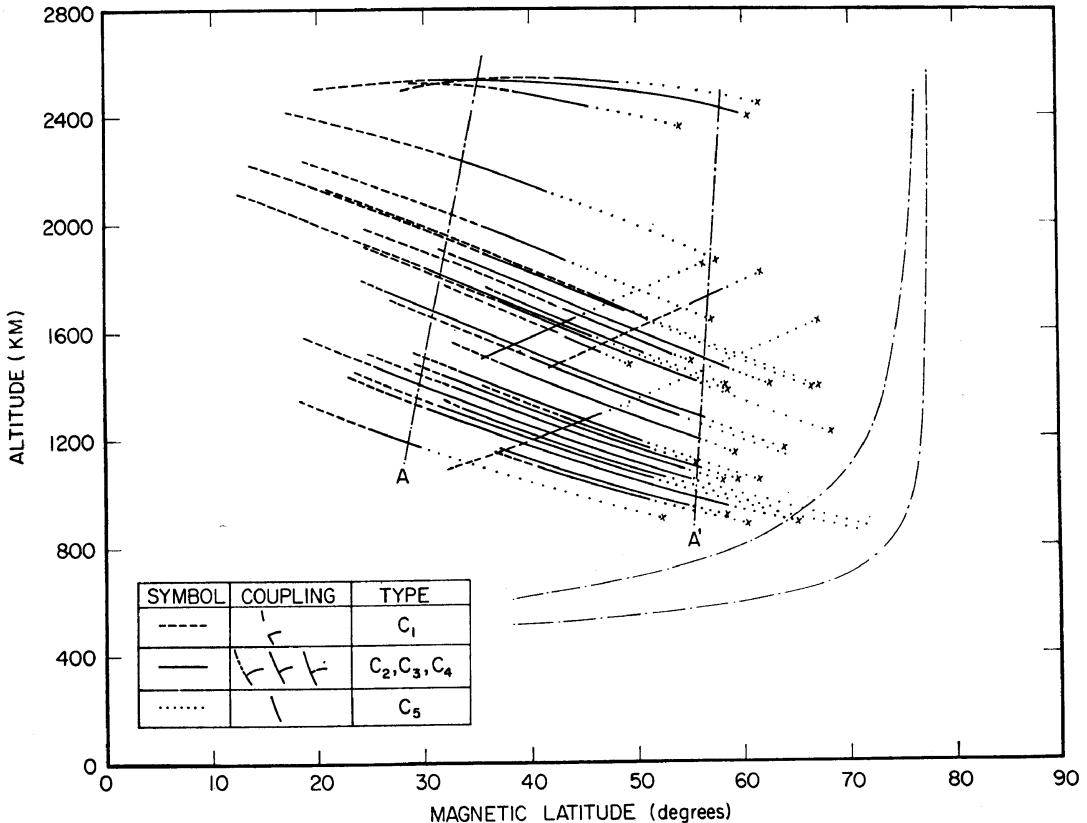


Fig. 7. Observed transition latitudes ($A - A'$) in coupling types compared with Jones' plot.

comparison, the black dots corresponding to measurements from individual whistler spectrograms and the dashed lines corresponding to the variation of Ω_c during low latitude portions of 7 passes. The pass in Figure 5 is a case where a cutoff frequency of $\Omega_c \approx 500$ Hz is observed from about 26° to 38° geomagnetic latitude.

DISCUSSION OF RESULTS

A comparison of these observed results with the coupling predictions of Jones shows general agreement as to the latitudinal sequence of coupling types. However, several significant discrepancies are evident: First, the observed latitudinal transitions of coupling types, as given in Figure 7, are at much lower latitudes (35° to 55° geomagnetic latitude) than predicted by Jones' calculations. Second, the latitudinal width of the transition from coupling types C_1 to C_2 is much larger than predicted by Jones' calculations. Third, the transition from one coupling type to another is often poorly defined with several different coupling types occurring at the same latitude. Fourth, partially formed proton whistlers of the type described by Jones (with a cutoff midway between the crossover frequency and the proton gyrofrequency) are rarely observed. Fifth, the observed electron whistler cutoff frequencies Ω_c show a large scatter about the curve predicted by Jones.

These disagreements, particularly in regard to the latitudinal transition of coupling types, indicates that significant differences must exist between the model used by Jones and the actual whistler propagation in the ionosphere. Some of the essential elements of Jones' calculations that are questionable are discussed below.

A. *The wave-normal angle.* In computing the transition latitudes shown in Figure 3, Jones assumed that the wave normal of a whistler is refracted to nearly vertical within the ionosphere. Although this assumption would appear to be reasonable at mid-latitudes where only small latitudinal gradients occur in the ionosphere, no specific experimental data is yet available to support this assumption. On the contrary, if the critical coupling angle θ_c determines the electron whistler cutoff frequency Ω_c , as proposed by Jones, the large scatter of cutoffs shown in Figure 8 indicate that a wide distribution in wave-normal angles is possible.

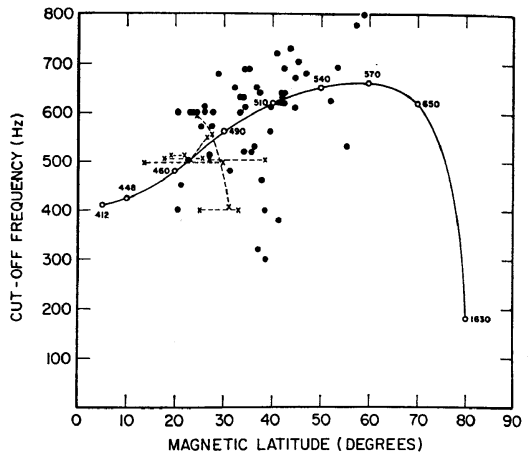


Fig. 8. Observed scatter of electron whistler cutoff frequency Ω_c vs. latitude compared with Jones' (solid line) calculated dependence of Ω_c on latitude.

A spread in the wave-normal angle distributions of electron whistlers can be expected if the lower boundary of the ionosphere is allowed to have time-dependent density and orientation variations. Upgoing whistlers would then be refracted to angles other than the vertical. Traveling ionospheric disturbances and ionospheric electron concentration irregularities, such as those discussed by Dyson [1967, 1969] and Heisler [1967], might be sources of such variations. A distribution of lightning sources spread out over a large latitude range below the satellite would also contribute, somewhat, to a distribution of wave-normal angles about the vertical.

B. *The critical coupling angle.* If the wave-normal angle is vertical and the transition in coupling types is controlled by the critical coupling angle, as suggested by Figure 2, then the observed transition latitudes imply a typical critical coupling angle of 25° to 30° , rather than 10° or less as computed by Gurnett et al. and Jones. Of the ionosphere parameters used to calculate the critical coupling angle, the collision frequency is probably the most uncertain. At high altitudes (~ 2500 km) it would be necessary to increase the collision frequencies by a factor of 100 or more above those used by Jones to obtain a critical coupling angle of 25° . Since such large collision frequencies ($\nu \approx 10 \text{ sec}^{-1}$) are not evident in the damping of proton whistlers [Gurnett and Brice, 1966] this explanation is

not considered possible. These considerations would suggest, therefore, that the critical coupling angle may not be the primary factor controlling the mode coupling of proton whistlers. The observed sharpness of the cutoff frequency Ω_c indicates, instead, that the primary function of the critical coupling angle is to determine which wave-normal angles undergo polarization reversal.

C. *The relative importance of vertical gradients and the critical coupling condition.* Referring to equation 2, it is seen that the coupling parameter ψ can become large either due to rapid changes in the polarization with respect to height [$\rho' = (1/K) \partial\rho/\partial z$] or due to the polarization approaching the critical coupling condition, $\rho = \pm 1$. Referring to Figure 2, it can be seen that the wave-normal angle must be very close (within one or two degrees) of the critical-coupling angle for the critical-coupling condition to play an important role in making ψ large. Since important coupling effects evidently occur for wave-normal angles considerably greater than the critical-coupling angle, it appears that vertical gradients in the polarization such as have been considered by Wang [1970] must be the dominant condition determining the mode coupling of electron and ion whistlers. Further evidence for the importance of ρ' is provided by Figure 2 which shows that, for paths like $E - E'$ that do not even come close to the critical-coupling point $\rho = +1$, the polarization changes very rapidly with altitude. For example, along path $E - E'$ the polarization changes from $\rho = 1.9 - i 0.8$ to $\rho = 1.8 + i 1.2$ in a distance of just 1 km (from 771.5 to 772.5 km), which is much less than a typical wavelength (~ 10 km) at that altitude and frequency. Thus, it would appear that detailed predictions of whistler mode coupling cannot be made from Figure 2 simply from how close the polarization trajectory approaches the critical-coupling condition $\rho = +1$ as has been done by Jones.

Further, more quantitative evidence emphasizing the importance of vertical gradients for producing the observed coupling is provided by Wang [1970] who has computed full wave solutions for the ionospheric model shown in Figure 1. The transmission coefficients computed by Wang show a latitudinal dependence which agrees very well with the observed lati-

tudinal dependence of coupling types shown in Figure 7. Since Wang's model considered a completely collisionless medium the contribution to the coupling parameter ψ is entirely attributable to the rapid change in the polarization with respect to height.

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