

The Low-Frequency Cutoff of ELF Emissions

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ELF and VLF radio noises observed by satellites in the ionosphere often have a very sharp lower cutoff frequency near the proton gyrofrequency. This paper summarizes the experimentally observed characteristics of this low-frequency cutoff and proposes an explanation for the cutoff based on the reflection of downward-propagating, extraordinary mode, waves near the two-ion cutoff frequency between the proton and helium gyrofrequencies. This explanation, if correct, provides the first direct evidence that chorus and ELF hiss emissions are generated at high altitudes (above 3000 km) and *not* near the base of the ionosphere. Ground-based observations of 700-Hz noise bands near the auroral zone, previously attributable to proton cyclotron radiation at low altitudes in the ionosphere, can now be explained by this reflection mechanism. Other possibly related effects (such as multiple ELF noise bands and the reflection of whistlers at the two-ion cutoff frequency) are discussed.

INTRODUCTION

Satellite observations of ELF and VLF radio noise have revealed that noise bands in the frequency range from a few hundred Hertz to several kilohertz often have a very sharp lower cutoff frequency near the proton gyrofrequency [Burns, 1966; Smith *et al.*, 1968; Guthart *et al.*, 1968]. In this paper we summarize a study of this noise band cutoff using data from the Injun 3 satellite and propose an explanation for the cutoff based on the reflection of downward-propagating, extraordinary mode waves near the two-ion cutoff between the proton and helium (or oxygen) gyrofrequencies. This explanation, together with the observed altitude dependence of the cutoff frequency, provides the first direct evidence that chorus and ELF hiss emissions are generated at high altitudes in the magnetosphere (above 3000 km) and *not* near the base of the ionosphere. The results of this study further indicate that ion-related propagation effects strongly influence the transmission of magnetospheric ELF radio noises to the ground and can prevent these radio noises from reaching the ground.

Ground observations of strong band emissions at approximately the proton gyrofrequency near the base of the ionosphere [Aarons *et al.*, 1960; Gustafsson *et al.*, 1960; Egeland *et al.*,

1965*a, b*], previously speculated to be proton cyclotron radiation, can now be interpreted as being due to a combination of the low-frequency cutoff in the transmission of ELF radio noise to the ground and the frequency spectrum of the emitted radiation.

CHARACTERISTICS OF THE LOW-FREQUENCY CUTOFF OF ELF EMISSIONS

In a previous study of VLF emissions by Taylor and Gurnett [1968], using data from the low-altitude (237–2735 km) Injun 3 satellite, it was found that the region of maximum occurrence and intensity of VLF radio noises from a few hundred Hz to about 7.0 kHz occurred during the local day, 0600–1800 magnetic local time (MLT) and about 55° to 75° invariant latitude (INV), with a broad maximum from about 9 to 11 hours MLT and from 60° to 70° INV. (MLT is the hour angle between the magnetic meridian through the satellite and the magnetic meridian through the sun, using the centered dipole approximation [Chamberlain, 1961]; and INV is $\arccos L^{-1/2}$ where L is McIlwain's [1961] geomagnetic shell parameter.)

The most common type of radio noise found in this region of maximum occurrence consisted of hiss (band-limited incoherent noise [Gallet, 1959; Helliwell, 1965]) in the frequency range from a few hundred Hertz up to about 2 kHz. This type of radio noise was called *ELF hiss*.

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Typical frequency-time spectrograms of ELF hiss observed by Injun 3 are shown in Figure 1.

Often the frequency spectrum of ELF hiss has a very sharply defined lower frequency limit ranging from about 300 to 700 Hz. Examples of ELF hiss illustrating this sharp low frequency cutoff are shown in Figures 1 (a), (b), and (c). The low-frequency cutoffs illustrated in Figure 1 have typical attenuations exceeding 20 db (roughly black to white on the spectrograms) in a 50-Hz frequency range. Since it is difficult to be quantitatively precise about the definition of a cutoff, we shall use the cutoffs illustrated in Figure 1 as being typical of what we mean by a cutoff in the noise spectrum.

The low-frequency cutoffs of the type illustrated in Figure 1 are not due to an instrumental effect, as is evidenced by the facts that (1) the cutoff frequency changes systematically with spatial position of the satellite, sometimes by several hundred Hertz during a 15-minute pass; (2) the same cutoff frequency has been observed for data received by two different telemetry

receiving stations; (3) the frequency response of the satellite-borne VLF receiver is not nearly sharp enough to account for the observed cutoff; and (4) the same cutoff effect has been observed by VLF receivers on the Alouette satellites (R. E. Barrington, personal communication) and the OGO satellites [Smith *et al.*, 1968; Guthart *et al.*, 1968].

The sharp low-frequency cutoff is *not* always observed for ELF hiss received with Injun 3. In some cases the absence of a clearly identifiable cutoff is due to the poor signal-to-noise ratio of the data below about 500 Hz because of the rapidly increasing attenuation of the Injun 3 VLF receiver in this frequency range. (See Gurnett and O'Brien [1964] for details of the experiment.) However, in many cases with good signal-to-noise ratios, the low-frequency cutoff can be seen to change from very sharp to diffuse or nonexistent in a time scale of the order of minutes. Approximately one-third of the ELF hiss events observed with Injun 3 have a recognizable low-frequency cutoff. This

NOISE BANDS WITH $57.5 < \text{INV} < 59.5$

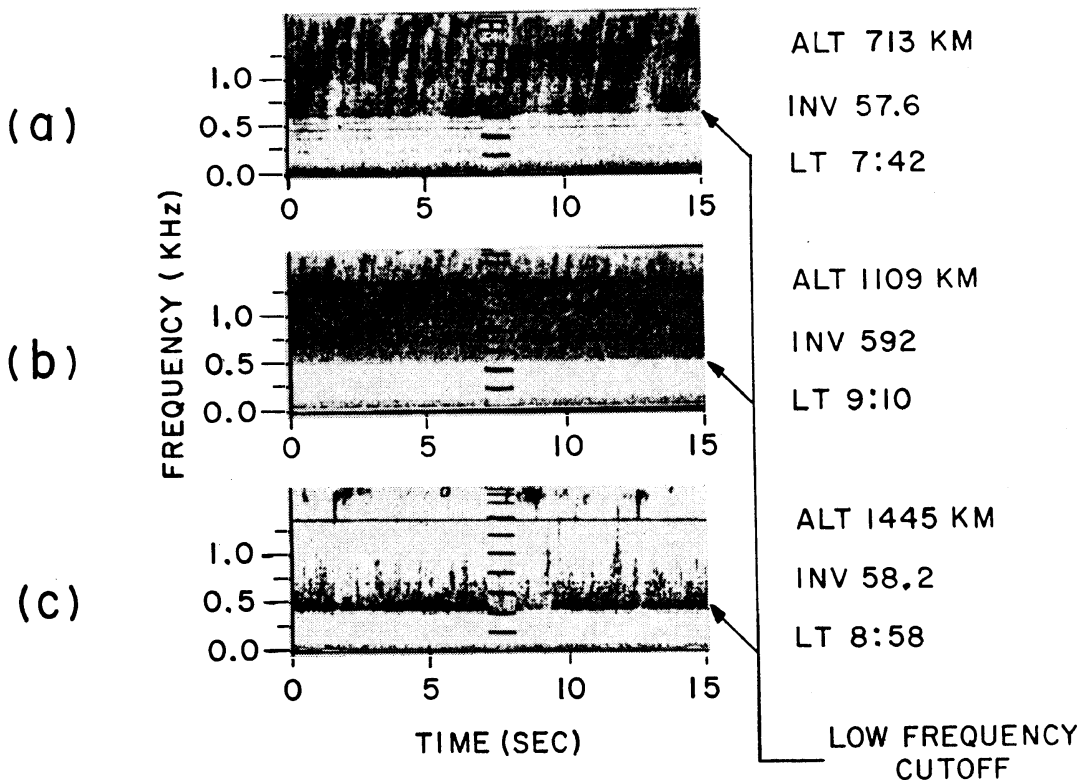


Fig. 1. Spectrograms illustrating the low-frequency cutoff of ELF emissions.

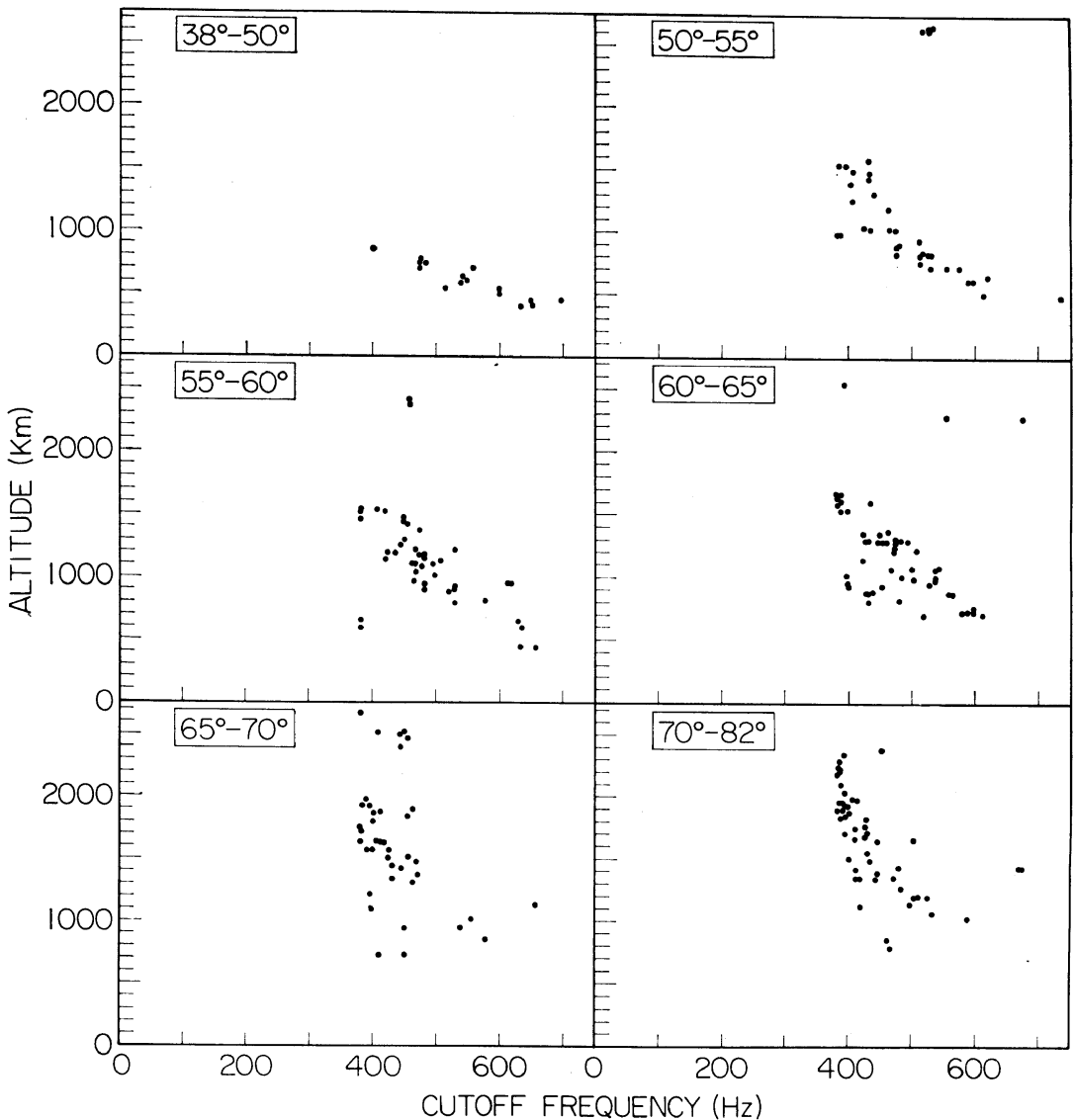


Fig. 2. Scatter plot of the lower cutoff frequency of ELF emissions as a function of altitude and latitude.

percentage of occurrence must be considered very uncertain because of the signal-to-noise ratio difficulty discussed above.

The low-frequency cutoff commonly observed for ELF hiss is also observed for other, less common, types of ELF emissions. Figures 1(a) and (c) illustrate the same low-frequency cutoff for discrete VLF emissions of the type called polar chorus. The low-frequency cutoff has also been observed for periodic emissions.

The cutoff frequency of ELF emissions, when it occurs, is found to decrease systematically

with increasing altitude and is usually less than the proton gyrofrequency at the satellite. Figure 1 shows examples of the low-frequency cutoff for three different altitudes at about the same latitude (57.5° – 59.5° INV) and illustrates the general tendency of the cutoff frequency to decrease with increasing altitude.

To provide some statistical evidence of the altitude dependence of the cutoff frequency, several hundred measurements of the cutoff frequency were made at various altitudes and latitudes. The cutoff frequencies measured are

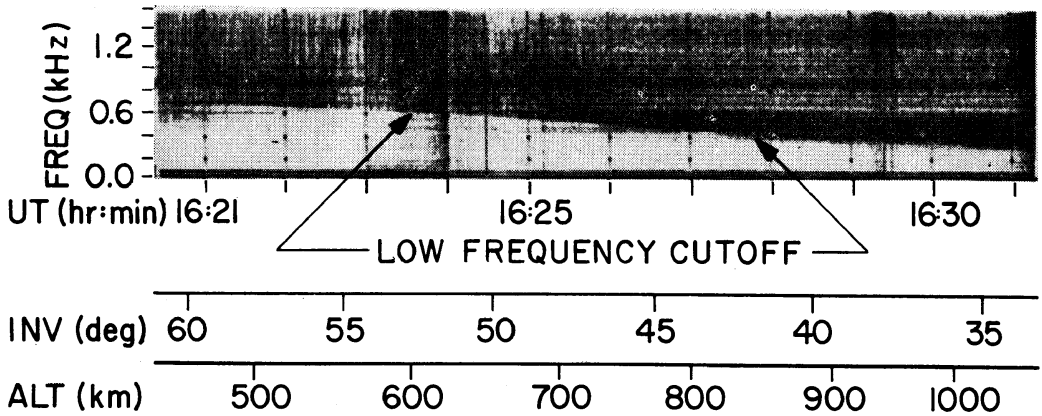


Fig. 3. Spectrogram showing the low-frequency cutoff variations with latitude and altitude for an individual pass.

shown as a function of altitude in Figure 2 for six latitude ranges. No measurements were made below about 380 Hz because of the poor signal-to-noise ratio and the uncertainty in identifying cutoffs at these frequencies. At the lower latitudes, 38° – 60° INV, the low-frequency cutoff is seen to decrease systematically with increasing altitude with only a small amount of scatter. All but about 10% of the cutoff frequencies (F_c) are within the range $0.8\Omega_p < F_c < \Omega_p$ (Ω_p = proton gyrofrequency). Significantly, perhaps, some of the cutoff frequencies are definitely above the proton gyrofrequency. At higher latitudes, particularly in the 65° – 70° invariant latitude range, the scatter increases considerably, and the cutoff frequency dependence on altitude is less well defined.

To confirm and possibly clarify the dependence suggested by the statistical study, several individual passes with ELF hiss were selected for analysis. These passes were selected to have continuous good quality ELF hiss data with a low-frequency cutoff covering the altitude and latitude ranges of interest (300–3000 km, and 40° – 70° INV). Figure 3 shows the frequency-time spectrogram for one of the individual passes studied. A continuous, sharply defined, low-frequency cutoff can be seen for the duration of this pass. Figure 4 illustrates the variation of the cutoff frequency (F_c) and its relationship to the proton gyrofrequency (Ω_p) during the pass. It is seen that near the beginning of the pass at low altitude (400 km) and high latitude (65° INV) the cutoff frequency is very close to the proton gyrofrequency. As the satellite

proceeds to higher altitudes and lower latitudes, the cutoff frequency drops significantly below the proton gyrofrequency until, near the end of the pass at about 1000-km altitude and 35° INV, the ratio of the cutoff frequency to the proton gyrofrequency is about 0.8. Analysis of other individual passes selected for study generally support the altitude dependence illustrated in Figure 4, namely that the cutoff frequency and the ratio of the cutoff frequency to the proton gyrofrequency (F_c/Ω_p) decrease with increasing altitude.

AN EXPLANATION OF THE LOW-FREQUENCY CUTOFF

One of the most important features of the low-frequency cutoff is the systematic decrease in the cutoff frequency with increasing altitude (Figure 2). This altitude dependence indicates that waves with frequencies just above the cutoff frequency at some given altitude are *not*

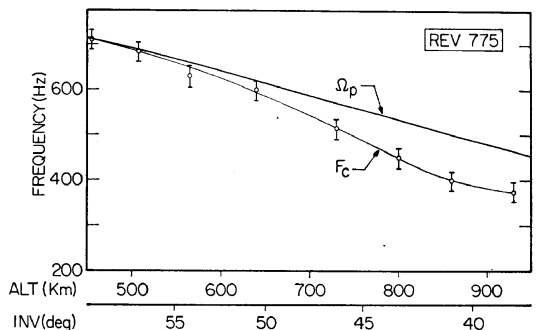


Fig. 4. Comparison of the low-frequency cutoff with the proton gyrofrequency.

observed at a lower altitude. If we consider that the waves are propagating in a horizontally stratified ionosphere, then we are led to two general possibilities for explaining this altitude dependence: (1) if the waves are down-coming from a source at a higher altitude, then the waves are being reflected (or absorbed) at the cutoff frequency, or (2) if the waves are entirely up-going, then they are being generated at the cutoff frequency. The first possibility (1) above is strictly a propagation effect, and the second possibility (2) involves the generation mechanism.

When the effects of ions are considered on the propagation of ELF waves in the ionosphere, a ready explanation arises for the observed cutoff. The propagation of electromagnetic waves in the ionosphere at frequencies of the order of the ion gyrofrequencies has been discussed by *Gurnett et al.* [1965] in connection with ion cyclotron whistlers. One of the results of principal interest to this paper is a cutoff in the extraordinary mode of propagation (corre-

sponding to the usual whistler mode when ions are not considered) at a frequency between the proton and helium gyrofrequencies. This cutoff frequency is called the $L = 0$ cutoff frequency [Stix, 1962] or the two-ion cutoff frequency [Smith and Brice, 1964]. At the $L = 0$ cutoff frequency, the index of refraction goes to zero for all angles of propagation, and the extraordinary mode becomes evanescent (non-propagating).

The importance of the $L = 0$ cutoff frequency for the propagation of ELF waves in the ionosphere can be best illustrated using the plot of various critical frequencies versus altitude shown in Figure 5. The fractional concentrations of H^+ , He^+ , and O^+ , shown at the bottom of Figure 5, are typical of a midlatitude, local night (temperature = 800°K) ionosphere and are identical to the model ionosphere used in *Gurnett et al.* [1965]. The critical frequencies plotted in Figure 5 are the proton gyrofrequency (Ω_p), the crossover frequency [Smith and Brice, 1964] (also labeled $D = 0$ according to the nomen-

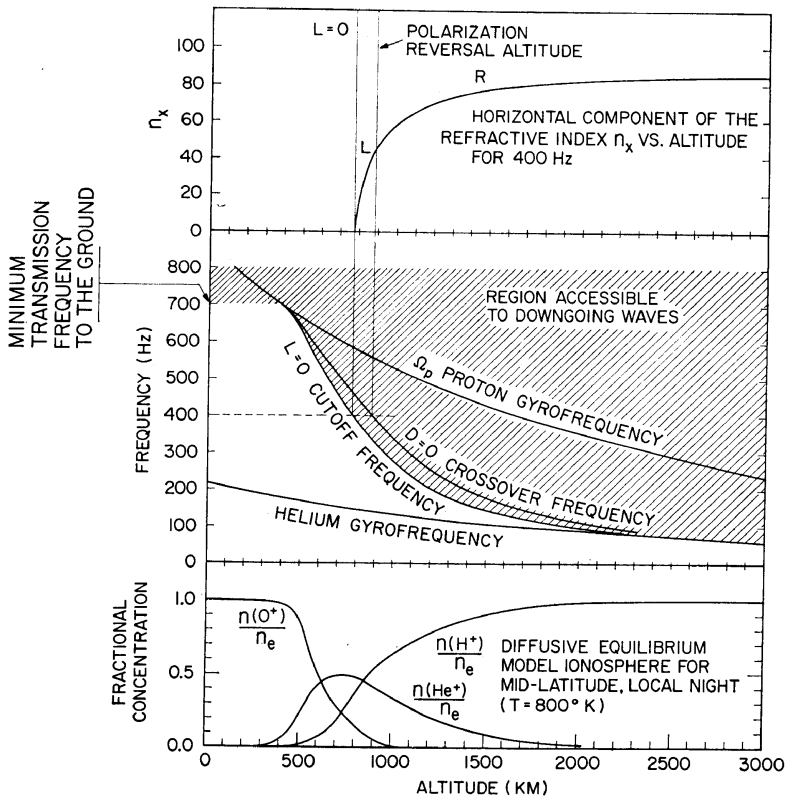


Fig. 5. Critical frequencies and horizontal index of refraction versus altitude for a model ionosphere.

clature of *Stix* [1962]), and the $L = 0$ cutoff frequency.

The role that these critical frequencies play for ELF waves propagating in the ionosphere can be illustrated by following a wave propagating downward from a source at high altitudes. Starting at a high altitude of 3000 km and a representative frequency of 400 Hz the wave must be propagating in the extraordinary (whistlers) mode since above the proton gyrofrequency the ordinary mode is evanescent (up to frequencies of the order of the electron gyrofrequency, ~ 1.0 MHz). At this altitude, the extraordinary mode is right-hand polarized. As the wave propagates downward, no major effect occurs until it reaches the altitude where the wave frequency is equal to the crossover frequency (about 880-km altitude for 400 Hz in Figure 5). As the wave crosses the $D = 0$ (crossover frequency) altitude, the polarization changes from right-hand to left-hand. This polarization reversal effect occurs only for plasmas with two or more ions [*Stix*, 1962] and was first demonstrated to occur for proton whistlers propagating upward from the base of the ionosphere [*Gurnett et al.*, 1965]. After the polarization reversal, the wave can continue to propagate downward, left-hand polarized, until the altitude is reached at which the wave frequency is equal to the $L = 0$ cutoff frequency (about 780 km for 400 Hz in Figure 5). At this altitude the index of refraction for the extraordinary mode (now left-hand polarized) goes to zero for all angles of propagation and is imaginary (nonpropagating) at all lower altitudes. Thus, for waves propagating downward from a high-altitude source, only altitudes above the $L = 0$ altitudes are accessible to these waves. The region accessible to downward-propagating waves is illustrated by cross-hatching in Figure 5. The minimum transmission frequency to the ground (700 Hz in Figure 5) is determined by the altitude at which the H^+ concentration becomes so small that polarization reversal no longer occurs when collisions are considered [*Jones*, 1968]. This minimum transmission frequency produces an analogous cutoff for upward-propagating waves that is commonly observed for the right-hand polarized whistler preceding ion cyclotron whistlers [*Gurnett et al.*, 1965].

Because of refraction as the wave approaches

the $L = 0$ altitude, a downward-propagating wave will in general be reflected *before* it reaches the $L = 0$ altitude. The reflection altitude depends critically on the initial wave normal angle and can even be above the crossover or proton gyrofrequency altitudes. For a horizontally stratified ionosphere the altitude at which reflection takes place can be determined from a plot of the horizontal refractive index (n_x) as a function of altitude, as shown in the top of Figure 5. From Snell's law, reflection will take place when n_x is equal to the initial horizontal component of the refractive index vector ($n \sin \theta$, $\theta =$ initial angle of incidence).

From the plot of the horizontal index of refraction in Figure 5, it can be seen that the largest vertical gradient in n_x occurs in the altitude range between the $L = 0$ cutoff and the proton gyrofrequency altitudes. Thus, for a reasonably uniform distribution of initial wave normal angles, most of the waves will be reflected in this altitude range, or correspondingly, below the proton gyrofrequency and above the $L = 0$ cutoff frequency.

From this discussion of the effects of ions on downward-propagating ELF waves in the ionosphere, it is evident that the low-frequency cutoff of ELF emissions can be explained by the reflection of downward-propagating ELF emissions due to the large vertical gradient in the refractive index of the extraordinary mode near the $L = 0$ altitude. The following general points of agreement with the experimental data support this explanation.

Relation of the cutoff frequency to the proton gyrofrequency. The altitude of reflection and the corresponding cutoff frequency are expected to be generally below the proton gyrofrequency, as is generally observed. It is possible, however, with a sufficiently large initial wave normal angle, for the reflection altitude and the corresponding cutoff frequency to be above the proton gyrofrequency, as has been observed in a few cases.

Relation of the cutoff frequency to the $L = 0$ cutoff frequency. Calculations of the $L = 0$ cutoff frequency as a function of altitude for reasonable estimates of the ion concentrations generally show that the observed cutoff frequency is somewhat greater (10–20%) than the calculated $L = 0$ cutoff frequency. In a few cases at midlatitudes, where proton whistlers

are observed simultaneously with low frequency cutoff of ELF emissions, the observed cutoff frequency was found to be very near the crossover frequency, as determined from proton whistlers (see Figure 6). These observations are consistent with the explanation that the cutoff is due to the reflection of downgoing waves above the $L = 0$ altitude, and, therefore, *above* the $L = 0$ cutoff frequency. Since the cutoff frequency and the spectrum near the cutoff are strongly dependent on the initial wave normal angle, the observed cutoff is *not*, in general, the $L = 0$ cutoff frequency. Thus, the cutoff frequency cannot be easily used to obtain ion concentration information as has been done for the crossover frequency of ion cyclotron whistlers. The dependence of the frequency spectrum near the cutoff on the initial wave normal angle has been suggested (N. Brice, personal communication) as a method of determining the distribution of wave normal angles.

Altitude dependence of the cutoff frequency. Since the $L = 0$ cutoff frequency strongly influences the refractive index near the reflection altitude, the altitude dependence of the observed cutoff frequency is expected to be similar to the altitude dependence of the $L = 0$ cutoff frequency. As the $L = 0$ cutoff frequency and the ratio of the $L = 0$ cutoff frequency to the proton gyrofrequency always decreases with increasing altitude (see Figure 5), the general tendency for the cutoff frequency (F_c) and the ratio of the cutoff frequency to the proton gyrofrequency (F_c/Ω_p), to decrease with increasing altitude, as illustrated in Figures 2 and 4, is accounted for.

Transmission past the $L = 0$ cutoff and mode coupling. ELF emissions are often observed that extend considerably below the lower cutoff frequency of ELF hiss and below the $L = 0$ cutoff frequency estimated from reasonable models of the ion concentrations. These cases of transmission past the $L = 0$ cutoff seem to be particularly common at high latitudes (above 60° INV).

Transmission past the $L = 0$ cutoff can be readily explained by mode coupling near the crossover frequency, much as in the case of ion cyclotron whistlers [Gurnett *et al.*, 1965; Jones, 1968]. When the effects of collisions are included, a critical coupling angle (θ_c), relative

to the geomagnetic field, is obtained. For wave normal angles greater than θ_c , at the crossover frequency altitude, polarization reversal occurs in the usual way (right- to left-hand for downgoing waves), and the wave cannot go below the $L = 0$ altitude. For wave normal angles near θ_c , however, the phase velocities of the two modes are very nearly equal, and mode coupling is strong, with the result that both right- and left-hand polarized waves are produced below the crossover altitude. Since the $L = 0$ cutoff is only for left-hand polarized waves, the right-hand polarized component can be transmitted past the $L = 0$ cutoff frequency. For wave normal angles less than θ_c , polarization reversal does not occur [Jones, 1968], and all of the wave energy can be transmitted past the $L = 0$ altitude.

Since the critical coupling angle is usually rather small (5° - 10°), mode coupling effects are expected to occur only for waves propagating nearly parallel to the geomagnetic field at the crossover frequency altitude. Thus, mode coupling would tend to occur primarily for ducted propagation or for certain latitude ranges where the source, presumed to be near the equatorial plane, illuminates the ionosphere with wave normal angles nearly parallel to the geomagnetic field. Some of the observed cutoff characteristics, such as the tendency for sharply defined cutoffs and less scatter in the cutoff frequencies at low latitudes (less than 60° INV), appear to be consistent with the expected latitude variation in the wave normal angles from an ELF emission source near the equatorial plane at L values of 4 to 8. Considerable additional investigation is required to establish the role of mode coupling for ELF emissions observed at low altitudes in the ionosphere.

DISCUSSION

In addition to explaining the low-frequency cutoff of ELF emissions, the reflection of downgoing waves near the $L = 0$ cutoff frequency may have application to other ELF radio noise phenomena observed on the ground and by satellites. Possible effects related to the $L = 0$ cutoff frequency are discussed below.

700-Hz noise bands. ELF and VLF emissions in the frequency range from a few hundred Hertz to several kilohertz are very commonly observed from the ground at middle and high

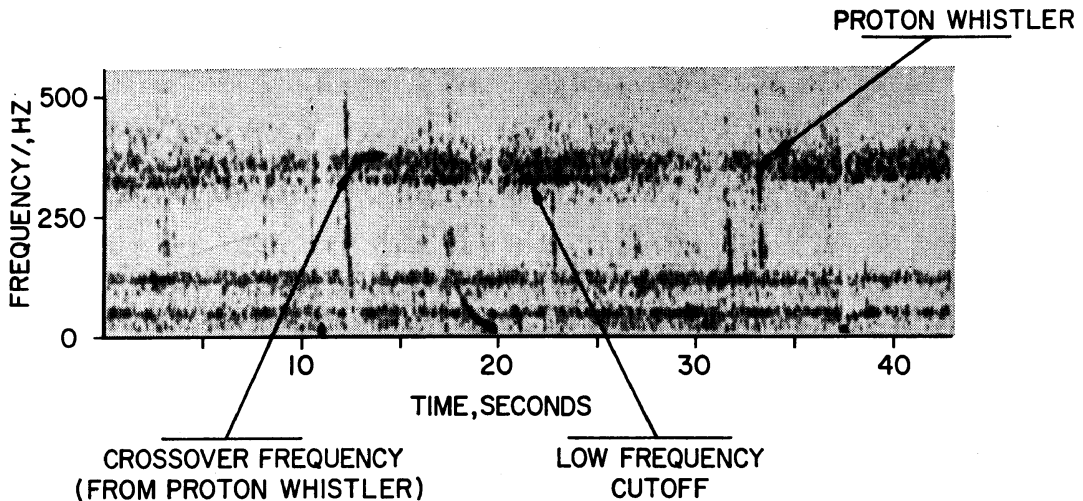
latitudes. Observations of ELF noise at Kiruna, Sweden, (65.3° geomagnetic latitude) by *Aarons et al.* [1960]; *Gustafsson et al.* [1960]; and *Egeland et al.* [1965a, b] have shown that the ELF noise spectrum generally has a strong peak at about 700 Hz. These observations of strong band emissions at approximately the proton gyrofrequency in the lower ionosphere (about 700 Hz at 400-km altitude) have led to the suggestion that this noise may be generated by proton cyclotron radiation in the ionosphere [*Aarons et al.*, 1960].

When the effects of ions are considered on the propagation of ELF waves, a fairly simple explanation arises for the 700-Hz noise band emissions observed by Aarons and others. From Figure 5 it is seen that the minimum transmission frequency to the ground is determined by the proton gyrofrequency at the base of the protonosphere. If the frequency spectrum of the down-going noise is increasing rapidly toward lower frequencies in this frequency range, as is often the case judging from the Injun 3 data, then the resulting ELF noise spectrum observed on the ground would be peaked near the minimum transmission frequency, the peak

being due to the combination of the sharp lower cutoff in the transmission to the ground and the frequency spectrum of the source. This explanation can account for the principal characteristics of the 700-Hz noise band given by *Egeland* [1965b], namely that (1) the peak noise intensity occurs at a frequency near the proton gyrofrequency in the lower ionosphere (about 700 Hz at 400-km altitude), (2) the noise spectrum has an asymmetrical shape, with a slope that is much steeper below the frequency of maximum amplitude than above, and (3) the noise band is relatively narrow (about 500 Hz).

Multiple noise bands. Figure 6 illustrates two examples of noise bands occurring at frequencies (200–300 Hz) considerably below the usual two-ion cutoff frequency near the proton gyrofrequency. These noise bands each have a sharp, low-frequency cutoff at approximately the He^+ gyrofrequency. Since there is also a two-ion cutoff frequency between the He^+ and O^+ gyrofrequencies, these low-frequency cutoffs may be due to the reflection of downgoing waves near the He^+-O^+ two-ion cutoff frequency, similar to the reflection of ELF emissions near

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Fig. 6. Simultaneous low-frequency cutoff and proton whistler illustrating the relationship of the cutoff frequency to the crossover frequency.

the H^+ - He^+ two-ion cutoff frequency. Many additional examples of multiple ELF noise bands must be studied to determine the role that ion effects have on the propagation of these noises.

Trapping of ELF waves in the ionosphere. For some ranges of initial wave normal angles and altitudes it is possible to trap ELF waves in the ionosphere near the $L = 0$ cutoff frequency. The mechanism for this trapping involves a low-altitude reflection point near the $L = 0$ altitude and a high-altitude reflection occurring because of the rapidly decreasing electron number density, hence refractive index, with increasing altitude. This wave trapping effect has been demonstrated with a ray tracing program [Gurnett and Shawhan, 1966]. No geophysical effect has yet been found that demonstrates the existence of such wave trapping in the ionosphere.

$L = 0$ cutoff effects for whistlers. Just as with ELF emissions, down-going whistlers in the ELF frequency range will be reflected above the $L = 0$ altitude in the absence of mode coupling. This type of reflection of whistlers has been observed in VLF data from the OGO 2 and 4 satellites [Muzzio, 1968].

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