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Fractional Concentration of Hydrogen Ions in the Ionosphere from VLF Proton Whistler Measurement

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Abstract. The derivation and the accuracy of an expression relating the fractional concentration of H^+, $\alpha_1 = n(H^+)/n_e$, to the crossover frequency ω_{12} for a VLF proton whistler are discussed. To an accuracy of ±3½% it is found that $\alpha_1 = (264/255)(1 - \Lambda_{12}^2)$, where $\Lambda_{12} = \omega_{12}/\Omega_1$ and Ω_1 is the proton gyrofrequency. Values of α_1 have been deduced from measurement of proton whistler spectrograms for some of the satellite Injun 3 VLF data and plotted against altitude (400–2600 km) and against invariant latitude (20°–63°) for summer daytime and winter nighttime of 1963. The fractional concentration of H^+ is found to be higher for winter nighttime than for summer daytime at all altitudes and latitudes; the ratio was approximately 3:1 at 1000 km. At 1000 km the value of α_1 dropped from 0.43 in the 20°–30° invariant latitude range to 0.27 in the 40°–45° range for summer daytime. For winter nighttime α_1 was nearly constant at 0.82 for 1000 km from 30° to 50° latitude, but dropped to 0.75 in the 50°–63° latitude range. Near 2400 km for summer daytime, α_1 drops from 0.65 at 46° latitude to 0.20 at 56° latitude. This same tendency is observed for winter nighttime, apparently being due to auroral-zone heating. These observations are consistent with the assumption that the heavier ions tend to predominate with increasing latitude for a given altitude. Comparison of these VLF proton whistler results for α_1 is made with reasonably good agreement to rocket ion mass spectrometer results of NASA 8.23 and to Alouette 1 VLF results deduced from lower hybrid resonance frequency. This general agreement establishes the VLF radio technique as an independent method for determining ion concentrations in the ionosphere. The general equations for uniquely determining the concentration of O^+, He^+, and H^+ from observation of the critical frequencies (crossovers, hybrid resonances, and cutoffs) due to the presence of ions are presented, and their usage is discussed.

1. INTRODUCTION

In a paper on proton whistlers by *Gurnett et al.* [1965] the theory of the proton whistler was presented and was supported quantitatively by VLF data from the Injun 3 satellite and by path length integrations based on a model ionosphere. The conclusion of that paper was that cold plasma magnetoionic theory was applicable in describing the experimental features of the proton whistler phenomenon (presented in more detail by *Shawhan* [1966]). The magnetoionic theory provides an expression that relates the fractional concentration of H^+ relative to the electron density at the satellite $\alpha_1 = n(H^+)/n_e$ to a measurable parameter of the proton whistler, the ratio of the crossover frequency ω_{12} to the proton gyrofrequency Ω_1 ($\Lambda_{12} = \omega_{12}/\Omega_1$). Using a small amount of data from Injun 3, altitude profiles of α_1 for local daytime and local nighttime were compared with α_1 profiles from model daytime and nighttime ionospheres with good agreement. The success of the magnetoionic theory in describing the proton whistler is reflected in this agreement between the model and the experimental α_1 altitude profiles and in the quantitative agreement between computed proton whistler travel times and experimental frequency-time spectrograms of proton whistlers. It is suggested by the paper of *Gurnett et al.* [1965], therefore, that VLF radio measurements by satellites and rockets at ion gyrofrequencies can provide valuable data on the seasonal, diurnal, and spatial concentration variations of the ionized constituents in the ionosphere.

In the present paper we are concerned in particular with the variation of the concentration ratio for H^+ $\alpha_1 = n(H^+)/n_e$ in the ionosphere as deduced from the measurement of Injun 3 VLF proton whistler spectrograms for summer daytime and winter nighttime of 1963. Section

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2 is a discussion of the derivation of the expression relating α_1 to the parameter $\Lambda_{12} = \omega_{12}/\Omega_1$ and its accuracy where ω_{12} is the crossover frequency measured from proton whistler spectrograms and Ω_1 is the proton gyrofrequency. The selection of proton whistler data from Injun 3 and the method of data analysis are explained in section 3. In section 4 the data are presented, and from them conclusions are drawn about the seasonal, diurnal, and spatial variation of α_1 for local summer daytime and local winter nighttime of 1963. The Injun 3 VLF measurements of α_1 are compared in section 5 with the values of α_1 derived using the VLF data on the lower hybrid resonance and the topside sounder electron density data from the Alouette 1 satellite [*Barrington et al.,* 1965], and with the values of α_1 calculated from rocket data (Argo-D4, Nasa 8.23) from a Bennett-type ion mass spectrometer of *Taylor et al.* [1963]. In section 6 the determination of the quantities $\alpha_2 = n(\mathrm{He}^+)/n_e$, $\alpha_3 = n(\mathrm{O}^+)/n_e$, and n_e from other VLF measurements is discussed.

2. Derivation and Accuracy of the Expression Relating α_1 and Λ_{12}

Since cold plasma magnetoionic theory has been shown to be substantially correct for the description of the proton whistler phenomenon [*Gurnett et al.,* 1965], we can use it to derive an expression which relates the fractional concentration of H$^+$ relative to the electron density $\alpha_1 = n(\mathrm{H}^+)/n_e$ to the ratio $\Lambda_{12} = \omega_{12}/\Omega_1$. As shown in Figure 1 of *Gurnett et al.* [1965] and in Figure 1 of *Shawhan* [1966], two frequencies can be measured from the frequency-time spectrogram of a proton whistler: the proton gyrofrequency and the crossover frequency. The value of the proton gyrofrequency Ω_1 can be measured with extremely good accuracy (to within 0.1%; see *Gurnett and Shawhan* [1966] or can be computed from the relation

$$\Omega_1 = eB/2\pi M_p c \qquad \text{cps (cgs units)}$$

For a multicomponent plasma *Smith and Brice* [1964] observed that a frequency exists between each ion gyrofrequency at which a wave propagating in an inhomogeneous plasma will change sense of polarization. These frequencies they called the crossover frequencies. *Gurnett* [1965] identified the frequency ω_{12} on proton whistler spectrograms as the crossover frequency. The abrupt change in the proton whistler travel time near ω_{12} occurs because of a rapid change in the group velocity near the crossover frequency. Using the notation of *Stix* [1962], and assuming a four-component plasma of electrons, H$^+$, He$^+$, and O$^+$, the crossover frequencies are obtained from the solution to the equation [*Gurnett et al.,* 1965, equation 20]

$$D = \frac{1\pi_e^{\ 2}}{\Lambda\Omega_1\Omega_e}\left[1 - \frac{\alpha_1}{1 - \Lambda^2} - \frac{\alpha_2}{1 - 16\Lambda^2} - \frac{\alpha_3}{1 - 256\Lambda^2}\right] = 0 \qquad (1)$$

where $\alpha_k = n(k)/n_e$. (Subscripts k are e, 1, 2, and 3 for electrons, H$^+$, He$^+$, and O$^+$, respectively.)

The two solutions to this equation are $\Lambda_{12} = \omega_{12}/\Omega_1$ for $\Omega_1 > \omega_{12} > \Omega_2$, and $\Lambda_{23} = \omega_{23}/\Omega_1$ for $\Omega_2 > \omega_{23} > \Omega_3$. Using the charge neutrality condition $\alpha_1 + \alpha_2 + \alpha_3 = 1$, equation 1 can be solved for α_1 and α_2 [*Gurnett,* 1965]

$$\alpha_1 = (16/15)(256/255) \cdot(1 - \Lambda_{12}^{\ 2})(1 - \Lambda_{23}^{\ 2}) \qquad (2)$$

$$\alpha_1 = (16/15)(16\Lambda_{12}^{\ 2} - 1)(1 - 16\Lambda_{23}^{\ 2}) \qquad (3)$$

Noting that, for the assumed plasma, Λ_{23} has the range $1/16 \leq \Lambda_{23} \leq 1/4$, we obtain the following inequality for α_1 in terms of Λ_{12}:

$$(16/15)(1 - \Lambda_{12}^{\ 2}) > \alpha_1 > (256/255)(1 - \Lambda_{12}^{\ 2}) \qquad (4)$$

By taking α_1 to be the average of the right- and left-hand sides of this inequality, α_1 can be estimated to within ±3½% given only Λ_{12}. To a good approximation, then, we can write

$$\alpha_1 = (264/255)(1 - \Lambda_{12}^{\ 2}) \qquad (5)$$

Equation 5 and the limits of equation 4 are plotted in Figure 1 to show the accuracy of the approximate relation between α_1 and Λ_{12}. For this paper equation 5 is used to calculate the concentration of H$^+$ relative to the electron density in the region of the satellite from measurement of the crossover frequency ω_{12} on proton whistler spectrograms.

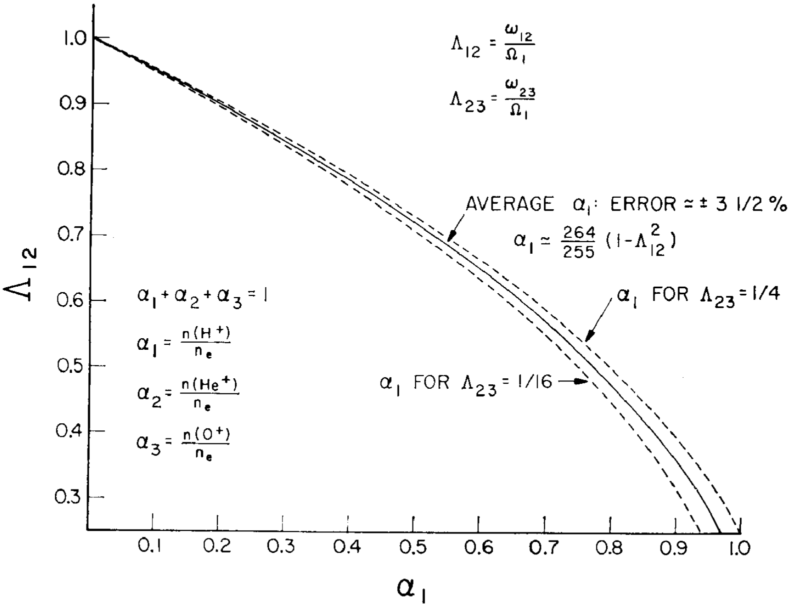

Fig. 1. Approximate relation between α_1 and Λ_{12} with limits indicated.

3. Selection and Method of Data Analysis

The discussion in the preceding section has demonstrated that the concentration of H^+ relative to the electron density (α_1) can be calculated to an accuracy of $\pm 3\frac{1}{2}\%$ from the measurement of the proton whistler crossover frequency. Therefore, values of α_1 can be determined by rockets and satellites in the regions of the ionosphere where proton whistlers occur, i.e., where H^+ is present and where whistlers occur. Experimental observations of proton whistlers in the Injun 3 VLF data are reported and discussed in detail by *Shawhan* [1966]. In summary, proton whistlers were observed to occur between approximately 440 km and apogee (2785 km) during local nighttime and 660 km and apogee during local daytime. In geographic latitude they were observed to occur between +63° and −40°.

It is also found that proton whistlers occur more frequently during local nighttime than during local daytime in the satellite altitude range (4:1). A higher frequency of occurrence is observed in the northern hemisphere than in the southern hemisphere (3:1). On a revolution during which proton whistlers occur, their occurrence is typically observed after every short fractional-hop whistler detectable at the satellite (up to 5 a minute). The spatial extent and frequency of proton whistler occurrence are attributable to the diurnal and seasonal variations in the H^+ concentration within the satellite orbit space and to the occurrence pattern of lightning discharges (the source of whistler energy) in the atmosphere. Within this region of the ionosphere, therefore, Injun 3 VLF data can provide an estimate of the quantity α_1.

All the Injun 3 VLF data were sorted by local summer and local winter and then by local daytime and local nighttime. For the northern hemisphere, winter was taken from December 21, 1962, to March 21, 1963, and summer from June 21, 1963, to September 21, 1963; for the southern hemisphere the winter and summer periods were reversed. Data from autumn and spring were excluded because of the seasonal changes over these 3-month periods. Corresponding to the flattest parts of the diurnal temperature curve, local daytime was chosen as 1100–1700 hours and local nighttime as 2300–0500 hours.

From these four graphs of data, summer daytime (SD), summer nighttime (SN), winter daytime (WD), and winter nighttime (WN), a sample of the data for summer daytime and winter nighttime was analyzed aurally. When proton whistlers were heard, approximately 1 each minute was rerecorded. The rerecorded proton whistlers were then displayed on a Sonograph high-resolution frequency-time spectrum analyzer (Kay Electric, Pine Brook, New Jersey). Measurements of the crossover frequency ω_{12} were made from these spectrograms to an estimated accuracy of ±3 cps. The value of the proton gyrofrequency Ω_1 was calculated from the magnitude of the geomagnetic field at the satellite using the Jensen-Cain spherical harmonic expansion for epoch 1960. By equation 5 these ratios $\Lambda_{12} = \omega_{12}/\Omega_1$ were converted to values of α_1. Of the 183 processed spectrograms, the proton whistler crossover frequency was measurable on 93.

In Figure 2 are plotted the 36 local summer daytime α_1 points that fell in an invariant latitude range of 20° to 56° (invariant latitude $= \arccos L^{-1/2}$, where L is McIlwain's parameter), and the 57 winter nighttime α_1 values in the range of 30° to 63° invariant latitude. These plots illustrate the compounded seasonal and diurnal variations of the α_1 versus altitude profiles. The error bars result from the inability to determine exactly the crossover frequency from the travel time of the proton whistler. Ideally, if the wave normal angle of the whistler is zero, the travel time to the satellite has a sharp discontinuity at the crossover frequency. For an actual proton whistler the wave normal angle is generally not zero, and the travel-time trace is rounded off near the crossover frequency, thus limiting the accuracy to which ω_{12} can be determined. It is estimated, however, that the measurement accuracy for determining α_1 is always less than about ±0.05.

In Figure 3 the α_1 values for summer day-

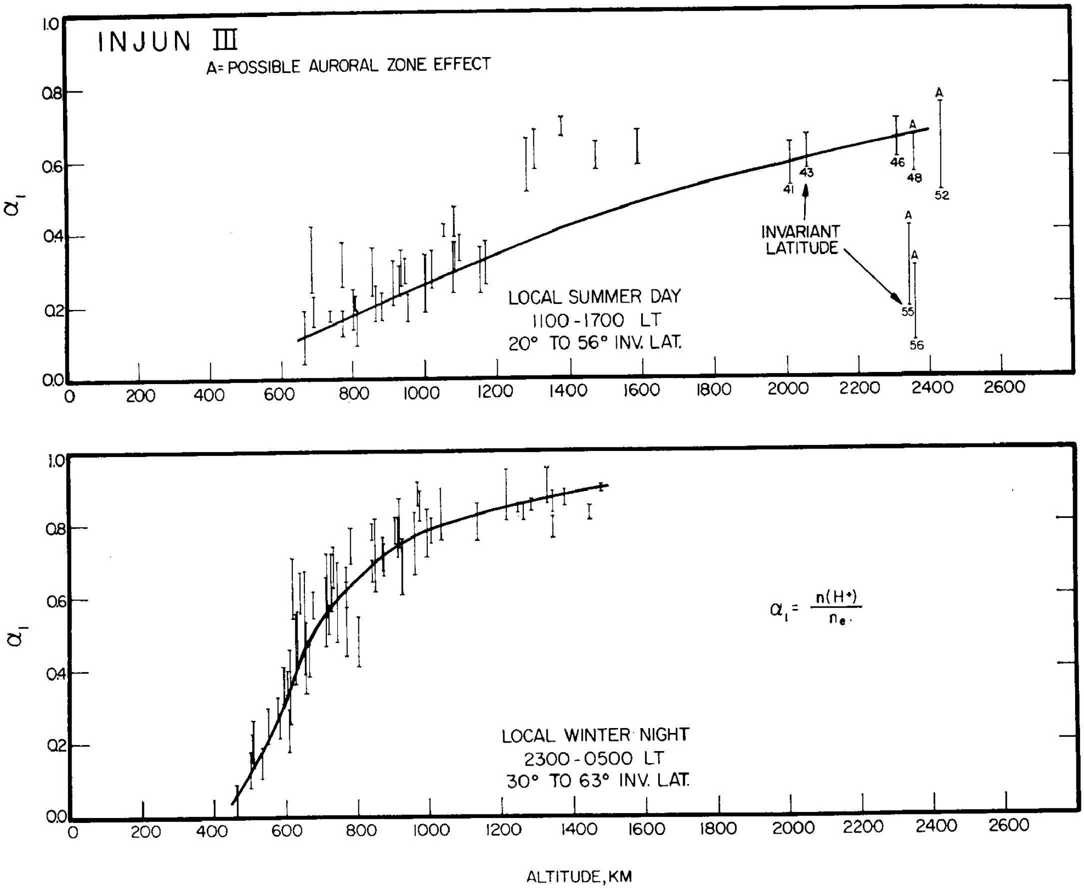

Fig. 2. α_1 versus altitude for summer daytime and winter nighttime. Error bars indicate inability to determine crossover frequency.

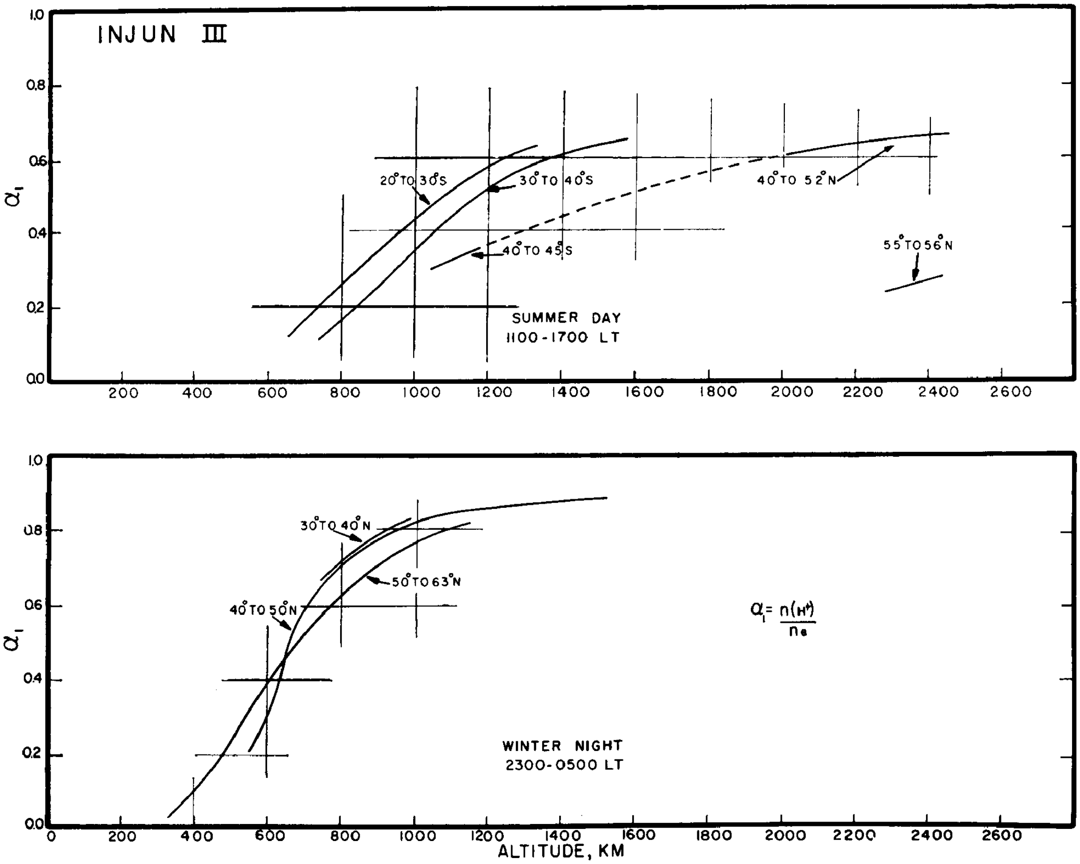

Fig. 3. Invariant latitude dependence of α_1 versus altitude for summer daytime and winter nighttime.

time and winter nighttime have been plotted against altitude according to the indicated invariant latitude intervals, and a smooth curve has been drawn through the points for each interval. Comparison of the α_1 altitude profiles for each latitude interval shows the invariant latitude variation of α_1 for summer daytime and for winter nighttime.

Figures 4 and 5 are another representation of these same data for summer daytime and winter nighttime. On these plots the center points of the error bars in Figure 2 are plotted as the percentage of H^+ in the altitude–invariant latitude space. The points in these plots were used to obtain the smooth curves for each invariant latitude range of Figure 3 and therefore illustrate the same general trends in altitude and latitude variations of α_1 for summer daytime and winter nighttime as Figures 2 and 3. The percentages of H^+ followed by the letter *S* indicate that these values were for the southern magnetic hemisphere, primarily from the region of the ionosphere over Woomera, Australia. Recordings over North America were the source of most of the remaining data, but a few points were taken from recordings at Winkfield, England. These Injun 3 proton whistler results for α_1 are compared with data from the rocket Nasa 8.23 in Figure 6 and from Alouette 1 in Figure 7.

The data presented in this paper are only a sample of the Injun 3 proton whistler data available for studying the variations in the concentration of H^+. Table 1, part A, lists the number of Injun 3 revolutions listened to, the number of revolutions on which proton whistlers were heard, the number of proton whistlers recorded, the number of good measurements of ω_{12}, and the ratio of good data points to the number of revolutions listened to, *R*, for sum-

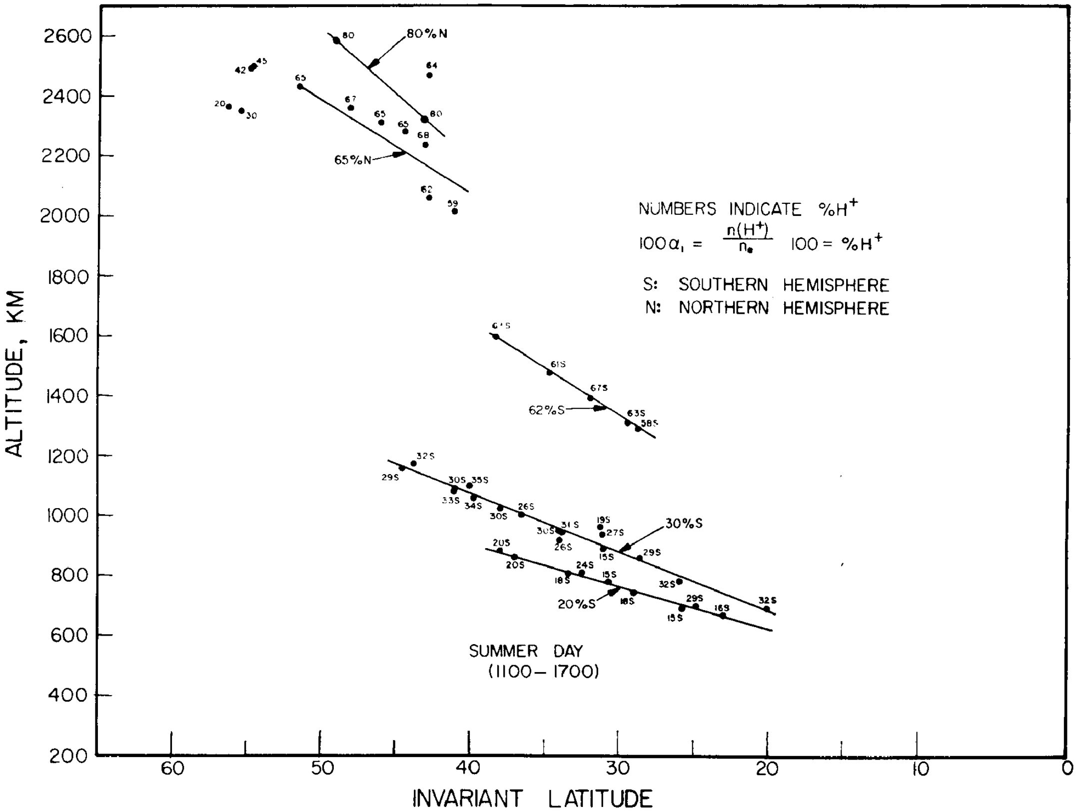

Fig. 4. Average percentage of H+ in altitude and invariant latitude for summer daytime.

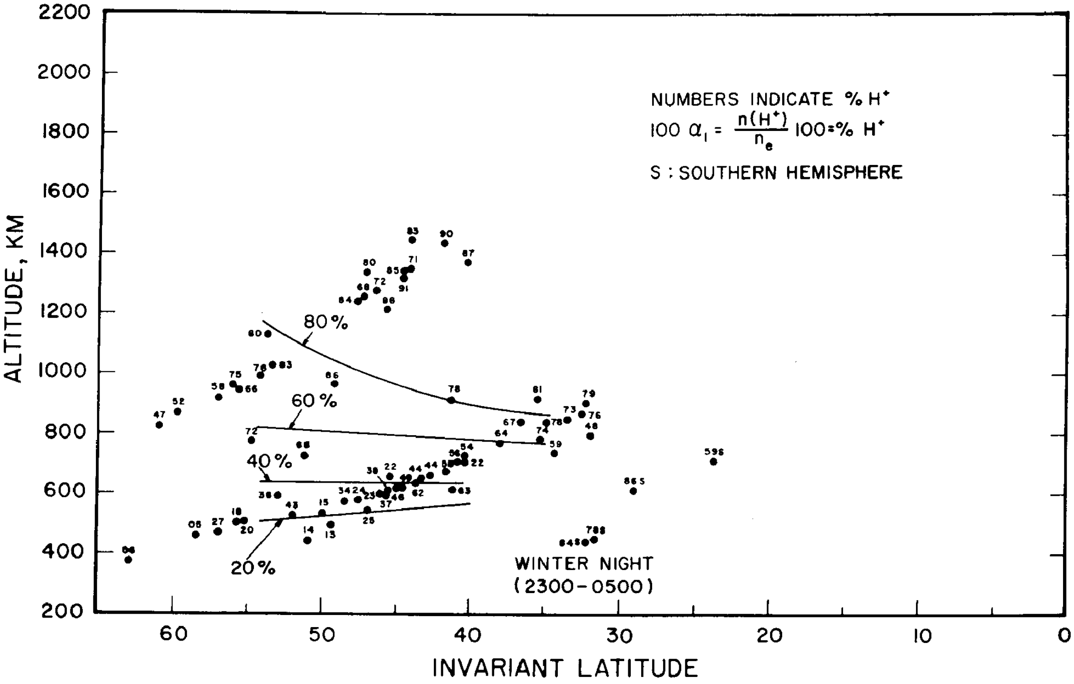

Fig. 5. Average percentage of H+ in altitude and invariant latitude for winter nighttime.

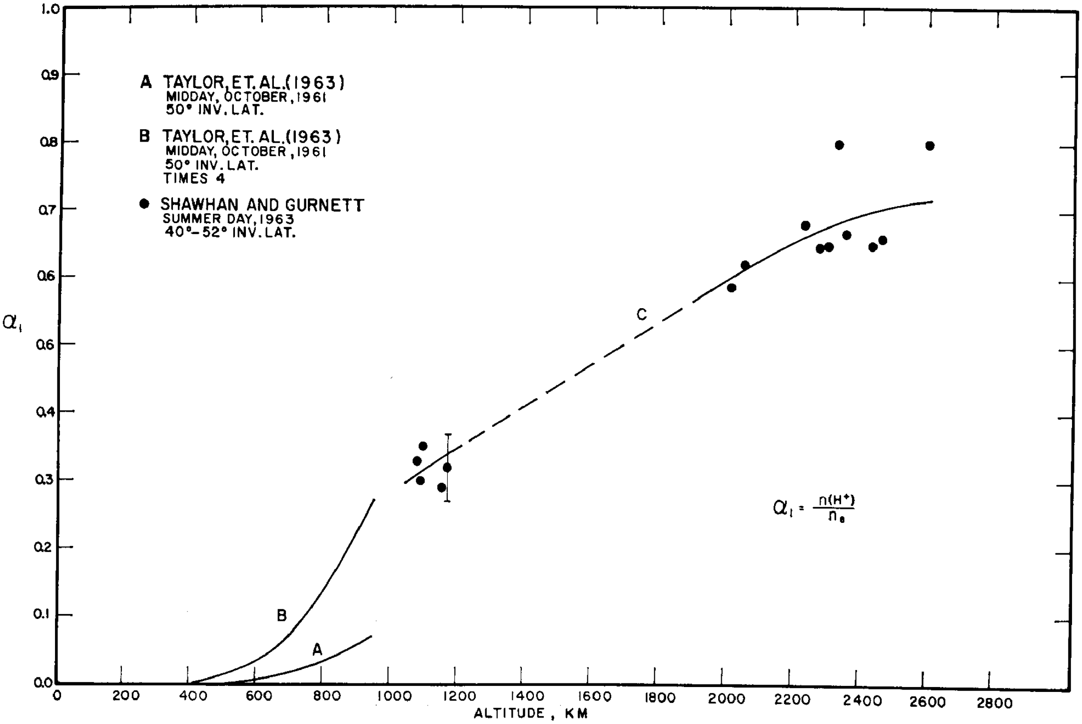

Fig. 6. Comparison of Injun 3 proton whistler VLF α_1 values with ion mass spectrometer values of rocket Nasa 8.23.

mer daytime (SD) and winter nighttime (WN) in the preliminary data study. These values of *R* are used in part B of the table to estimate the number of potential data points in the Injun 3 data from knowing the number of revolutions in each of the data groups (SD, SN, WD, and WN). Data analysis by a real time spectrum analyzer rather than by listening would provide all the proton whistlers for which the crossover frequency is measurable, and consequently the number of potential points given in 2*a* and *b*, part B, of the table can probably be increased by a factor of 2.

The table suggests the amount of data that can be obtained on the concentration of H^+ and of the other ionospheric constituents (discussed in section 6) from a satellite VLF experiment similar to the Injun 3 experiment which operated for about 10 months (see *Gurnett and O'Brien* [1964] for a complete description of the Injun 3 VLF experiment). With the spatial and temporal coverage of a satellite VLF experiment and the high frequency of occurrence of proton whistlers (up to 5 a minute), information can be obtained on the short-term concentration irregularities of the ionized constituents in time and space as well as on the longer-term diurnal and seasonable variations.

TABLE 1. Potential of Injun 3 Data

	SD	SN	WD	WN
A. Preliminary data				
1. No. revolutions listened to	67			40
2. No. revs with proton whistlers	15			26
3. No. proton whistlers recorded	63			123
4. No. good points	36			57
5. *R* = ratio of good points to no. revs listened to	0.54			1.42
B. Entire data				
1. No. revs possible	255	327	79	110
2. *R* times no. revs possible = no. potential good points				
a. Day, *R* = 0.538	137		43	
b. Night, *R* = 1.42		464		154

4. Conclusions from the Data

As discussed in section 3, the Injun 3 VLF data on the density of H^+ relative to the electron density are displayed as a function of altitude in Figure 2, of altitude for different latitude intervals in Figure 3, and of altitude and latitude in Figures 4 and 5 for summer daytime (SD) and winter nighttime (WN) of 1963. From these data four conclusions can be drawn:

A. The value of α_1 is lower for summer daytime than for winter nighttime at all latitudes and altitudes observed.

This compounded seasonal and diurnal effect can be seen most easily from Figure 2. All the summer daytime and winter nighttime data points are plotted, and a smooth solid curve is drawn through them. At a given altitude, say 1000 km, α_1 has a value of 0.25 for summer daytime and a value of 0.75 for winter nighttime. This higher concentration of H^+ for winter nighttime holds over the entire observed altitude range. The fact that this seasonal-diurnal difference between summer daytime and winter nighttime is observed at different invariant latitudes for a given altitude is shown in Figure 3. Again, a comparison of individual values of α_1 (%) in Figure 4 for summer daytime and Figure 5 for winter nighttime indicates that the summer daytime percentages of H^+ are less than the winter nighttime percentages at all altitudes and latitudes (a factor of approximately 1:3 at 1000 km).

B. The value of α_1 tends to decrease with increasing latitude for a given altitude. This decrease is greater for summer daytime than for winter nighttime.

At 1000 km in the summer daytime plot of Figure 3, α_1 has a value of approximately 0.43 for the 20°–30° south invariant latitude range, of 0.35 for 30°–40° south, and of 0.27 for 40°–45° south (extrapolated). For summer daytime at 1000 km, therefore, a negative gradient in α_1 of approximately 0.08 for 10° of invariant latitude exists in the range observed. Winter nighttime values of α_1 are nearly the same between 30° and 50° north invariant latitude ($\alpha_1 = 0.82$ at 1000 km). The value of α_1 has dropped, however, to 0.75 at 1000 km in the 50°–63° north invariant latitude range. For winter nighttime at 1000 km, therefore, a negative gradient in α_1 with invariant latitude is observed only above 50°.

This variation of α_1 with invariant latitude can be seen from a different viewpoint in Figures 4 and 5, which are plots of the data points in altitude and invariant latitude with each point marked by the average percentage of H^+ (centerpoint value of the error bars of Figure 2). In Figure 4, for the summer daytime values, contours of nearly equal percentages of H^+ have been sketched and labeled by the average percentage of the nearby points. These contours tend to be straight lines and to have steeper slopes with increasing percentage. Note that the contours for 65%N and 62% S do not match in altitude at 40° latitude, but they do have approximately the same slope; the mismatch is probably explained by different ionospheric conditions at the times of data acquisition. The contours reflect the same conclusions as were reached from Figure 3: picking a given altitude, α_1 decreases with latitude; picking a given latitude, α_1 increases with altitude. Figure 5 is a plot for the winter nighttime values of α_1 (%). Although the points are scattered so that representative constant percentage contours do not fit as well as for the summer daytime data, the same general trends can be seen for the variance of α_1 in altitude and latitude as for the summer daytime values.

C. At an altitude near 2400 km in the northern hemisphere for summer daytime, the value of α_1 drops from 0.65 at 46° invariant latitude to 0.20 at 56°.

For the mid-invariant latitudes it was found that α_1 gradually decreased with increasing latitude at a given altitude (conclusion B). In Figure 2 the error bars for the summer daytime plot are labeled by their invariant latitude values near 2400 km. The values of α_1 for the points above 46° latitude decrease markedly; α_1 changes from approximately 0.65 at 52° to approximately 0.20 at 56°, a ratio of more than 3:1 in only 4° of invariant latitude. To a lesser degree the winter nighttime values of α_1 tend to decrease above 50° invariant latitude. Near the altitude of 500 km in Figure 5, the concentration of H^+ changes from values ranging from 13 to 43% near 50° latitude to 5% at 58°. This same decreasing trend is evident between 800 and 1000 km for latitudes above 50°.

These marked decreases in α_1 with invariant latitude for summer daytime and winter nighttime seem to coincide with the auroral-zone region and are, therefore, probably related to auroral-zone heating by energetic precipitated particles.

D. The northern and southern hemisphere α_1 profiles tend to match when plotted on invariant latitude.

For summer daytime, data were observed in the same invariant latitude range for the northern hemisphere (40°–50°) and for the southern hemisphere (40°–45°). As is shown in Figure 3, a reasonably smooth curve can be drawn between these groups of data, indicating that H^+ is distributed continuously from the northern to the southern magnetic hemisphere for this invariant latitude range ($L \cong 2.0$). This observation supports the assumption that the ionospheric plasma is constrained to move along the geomagnetic field lines.

From these four experimental observations we have obtained a quantitative picture of the H^+ variations in the ionosphere for 1963 consistent with the accepted qualitative picture. During winter nighttime, at, say, 1000 km, the ionosphere is predominantly H^+ (82%) from the magnetic equator to approximately 50° invariant latitude ($L = 2.0$). Above 50° the percentage of H^+ drops to approximately 75% at 60°. During summer daytime at 1000 km, however, the percentage of H^+ drops from approximately 43% at 25° invariant latitude to 27% at 45° latitude. H^+ therefore appears to be the dominant ion near the magnetic equator, but at higher latitudes the heavier ions (O^+ and He^+) begin to dominate. At 2400 km and at 56° invariant latitude only 20% H^+ is observed during the summer daytime.

5. COMPARISON OF RESULTS WITH THOSE OF OTHER WORKERS

Although relatively few experimental measurements are available for comparison, the Injun 3 data treated in the previous section can be compared with the experimental data of the following two groups of workers: *Taylor et al.* [1963], and *Barrington et al.* [1965].

Taylor et al. flew a Bennett-type ion mass spectrometer and a cylindrical electrostatic probe to an altitude of 940 km in midday of October 1961 on an Argo-D4 rocket (Nasa 8.23) from Wallops Island, Virginia. The He^+ and H^+ concentrations were measured by the mass spectrometer, and the total ion concentration was measured by the electrostatic probe. An α_1 altitude profile calculated from their experimental results is presented as curve *A* of Figure 6. For comparison with the Injun 3 proton whistler data, the average values of α_1 for summer daytime of 1963 in the invariant latitude range 40°–52° are also plotted. This invariant latitude curve is used because Wallops Island lies at approximately 48.5° invariant latitude. Curve *A* and the Injun 3 values of α_1 are not in good agreement. Extrapolating curve *C* (drawn through the Injun 3 points) and curve *A* to 1000 km, the results of Taylor et al. indicate 8% H^+ whereas the Injun 3 results indicate 28%. This disagreement might be resolved by noting that the data were taken approximately a year and a half apart. Since the data of Taylor et al. preceded the solar minimum period and the Injun 3 data, we would expect qualitatively that the mass spectrometer values of α_1 would be lower. *Bauer* [1963] has suggested, however, that these results as reported by Taylor et al. do not support the generally accepted diffusive equilibrium model above 750 km. It has also been suggested that the results of Taylor's group for the H^+ concentration need to be increased by a factor of 4 (S. J. Bauer, private communication). Curve *B* of Figure 6 is four times curve *A*. Extrapolating curve *B* to 1000 km we find a value of approximately 32% for H^+, which is in quite good agreement with the Injun 3 results.

Barrington et al. [1965], with the Alouette 1 satellite, used a VLF phenomenon in conjunction with the topside sounder to set limits on the fractional abundance of the various ions. They assumed that a particular VLF noise band, observed with Alouette 1, was associated with the lower hybrid resonance (see *Stix*, [1962]). The lower cutoff frequency of this noise band is a function of the electron plasma frequency, the electron gyrofrequency, and the harmonic mean mass number for all the positive ions in the medium, m (eff). From the topside sounder the values of the electron gyrofrequency and plasma frequency were obtained. Knowing that m (eff) could range only between 1 and 16 amu (for O^+, He^+, H^+ ions),

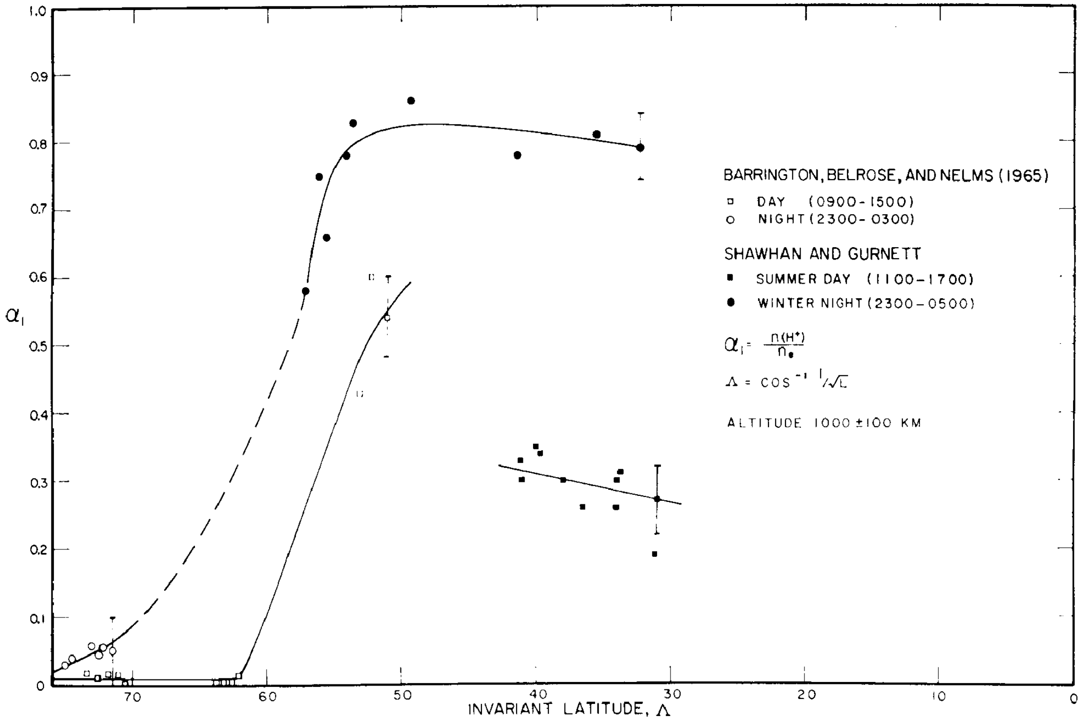

Fig. 7. Comparison of Injun 3 proton whistler VLF α_1 values with lower hybrid resonance VLF values of Alouette 1.

limits were set on the concentrations of these ions.

In Figure 7 the results of this experiment are compared with the Injun 3 proton whistler results at 1000 ± 100 km. The nighttime values for both sets of data are plotted by circles (see key in figure). Although no data were presented for the same invariant latitude, the nighttime curves seem to match quite well when extrapolated. Agreement between the Alouette 1 and Injun 3 data is not quite as good for the daytime near 50° invariant latitude. The Injun 3 data indicate a 35% H^+ concentration (extrapolated), whereas the Alouette 1 data indicate approximately 57%. Barrington et al. report that the data were taken over a 2-year period, so that it is possible that the data near 50° are for a winter daytime period. If so, we should expect their values of α_1 to be higher than for the Injun 3 summer daytime.

The general agreement of the Injun 3 and the Alouette 1 VLF determinations of the fractional concentration of H^+ (α_1) illustrates the self-consistency of deducing ion composition from VLF effects related to the presence of ions in the ionosphere. The general agreement of the Injun 3 VLF and the direct ion mass spectrometer determinations of α_1 therefore establish the VLF radio technique as an independent method for determining ion composition.

6. VLF Methods for Determining Fractional Concentration of He^+ and O^+

Data have been reported in this paper on the fractional concentration distribution of the H^+ ion (α_1) in the ionosphere which were deduced from measurement of the crossover frequency associated with the H^+ ion (between the H^+ and He^+ gyrofrequencies). The data of *Barrington et al.* [1965] were obtained from the VLF lower hybrid resonance and were used to set limits on the concentrations of the ions. Neither of these sets of VLF data, however, was able to uniquely specify the concentration of all three ions (O^+, He^+, H^+).

Smith and Brice [1964] and *Gurnett* [1965] have shown that three classes of critical frequencies are associated with the presence of ions in the ionosphere: the crossover frequencies, the hybrid resonance frequencies, and the cutoff frequencies (see also *Stix* [1962]). In the present paper we have used one of the crossover frequencies (associated with the H^+ and He^+ ions); *Barrington et al.* [1965] made use of the lower hybrid resonance frequency (associated with H^+ and the electrons). *Smith and Brice* [1964] have shown that the relative concentration of each of the ionic constituents can be determined from the knowledge of either all the crossover frequencies, all the ion-ion hybrid resonances, or all the multiple ion cutoff frequencies. *Gurnett* [1965] has shown that, for a plasma having three types of ions, a measurement of *any two* of the critical frequencies (crossovers, hybrid resonances, or cutoffs) uniquely determines the relative concentration α_1, α_2, and α_3 (since $\alpha_1 + \alpha_2 + \alpha_3 = 1$, we have only two independent parameters). Using the notation of *Stix* [1962], the critical frequencies are the solutions of the following equations.

(*a*) Crossover frequencies (X):

$$\frac{1}{1-\Lambda^2}\frac{255}{256}\alpha_1 + \frac{1}{1-16\Lambda^2}\frac{15}{16}\alpha_2 = 1 \qquad (6)$$

(*b*) Ion-ion hybrid frequencies (IH):

$$\frac{1+16\Lambda^2}{1-\Lambda^2}\frac{15}{16}\alpha_1 + \frac{1+64\Lambda^2}{1-16\Lambda^2}\frac{3}{4}\alpha_2 = 1 \qquad (7)$$

Lower hybrid resonance (LH):

$$15\alpha_1 + 3\alpha_2 = 16\frac{\Omega_{LH}^2}{\Omega_1\Omega_e}\left[1+\left(\frac{\Omega_e}{\pi_e}\right)^2\right] - 1 \qquad (8)$$

(*c*) Cutoff frequencies (C):

$$\frac{1}{1-\Lambda}\frac{15}{16}\alpha_1 + \frac{1}{1-4\Lambda}\frac{3}{4}\alpha_2 = 1 \qquad (9)$$

Given any two critical frequencies, the coefficients for two of the above equations are determined using $\Lambda = \omega_{\text{critical}}/\Omega_1$, and the two equations can be easily inverted to obtain α_1 and α_2. In principle, any of the following combinations can be used (X, X), (IH, IH), (C, C), (X, IH), (X, C), (IH, C), (X, LH), (IH, LH), (C, LH).

Observations of only two of the critical frequencies have been reported: the proton whistler crossover frequency and the lower hybrid resonance frequency. With simultaneous knowledge of the electron density (for π_e), the magnetic field strength for Ω_1 and Ω_e, and these two frequencies, equations 6 and 8 can be solved simultaneously for α_1 and α_2 (therefore α_3 also is determined). In another paper [*Gurnett and Shawhan*, 1966] we show how proton whistlers can be used to calculate both the electron density and the magnetic field strength. Thus, all the necessary information can be obtained from VLF data alone. Figure 8 is a plot that can be used to determine α_1 and α_2 by this method.

Although helium whistlers have not been positively identified in the Injun 3 VLF data because of poor frequency response near 100 cps, their existence has been predicted. A crossover frequency measurement of the helium whistler would also give the second piece of information needed to determine α_1, α_2, α_3 uniquely from equation 6.

Equations 6 through 9, therefore, are the relations needed to apply the VLF radio technique for determination of the ion concentrations in the upper ionosphere.

7. Conclusion

The derivation of the equation relating the crossover frequency of the proton whistler to the fractional concentration of H^+ at the satellite has been presented and discussed. It is found that, to an accuracy of ±3½%,

$$\alpha_1 = \frac{n(H^+)}{n_e} = \frac{264}{265}\left[1-\left(\frac{\omega_{12}}{\Omega_1}\right)^2\right]$$

where ω_{12} is the crossover frequency measured from proton whistler spectrograms and Ω_1 is the proton gyrofrequency. A preliminary analysis of Injun 3 VLF proton whistler data has led to the following conclusions about the variations of α_1 in altitude from approximately 400 km to 2600 km, and invariant latitude from approximately 20° to 63° for summer daytime and winter nighttime:

1. The value of α_1 is lower for summer daytime than for winter nighttime at all latitudes and altitudes observed.
2. The value of α_1 tends to decrease with increasing latitude for a given altitude. The decrease is greater for summer daytime than for winter nighttime.
3. At an altitude near 2400 km in the northern hemisphere for summer daytime, the value

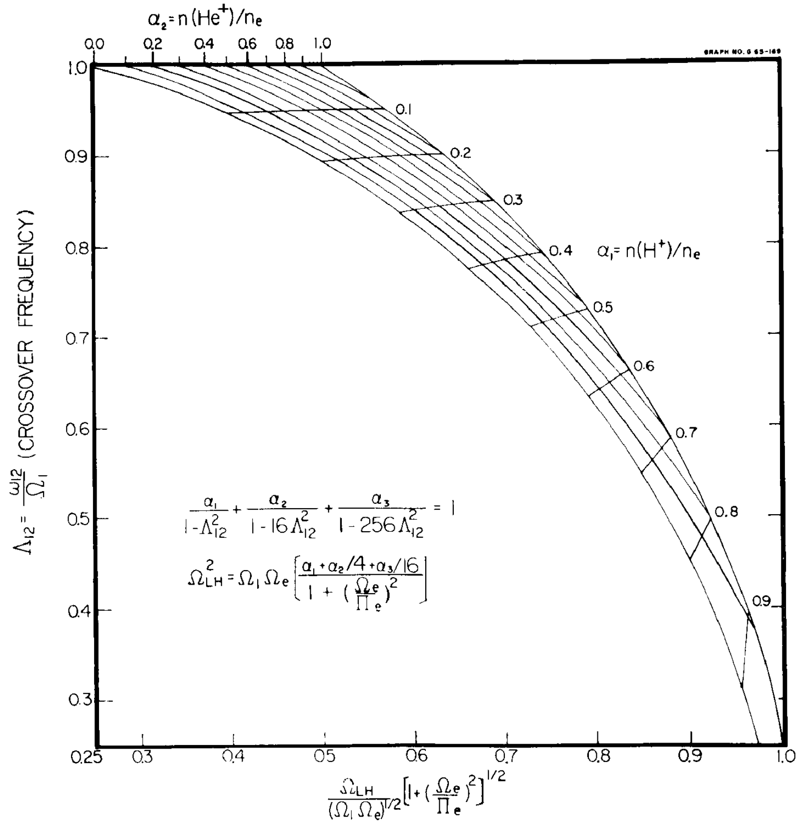

Fig. 8. Determination of ion concentrations from the proton crossover frequency, the lower hybrid resonance frequency, and the electron density.

of α_1 drops from 0.65 to 46° invariant latitude to 0.20 at 56°.

4. The northern and the southern hemisphere α_1 profiles tend to match when plotted by invariant latitude, suggesting that the plasma is constrained to move along geomagnetic field lines.

These observations are consistent with the assumption that the heavier ions (O^+ and He^+) tend to predominate at a given altitude for increasing latitude.

These data have been shown to be in reasonable agreement with direct rocket ion mass spectrometer measurements of H^+ by *Taylor et al.* [1963] (particularly with the 'corrected' curve, see section 5), and with measurements from another VLF phenomenon, the lower hybrid resonance, by *Barrington et al.* [1965] with Alouette 1. Equations were presented that enable VLF observations of ion critical frequencies (crossovers, resonances, and cutoffs) to be used to deduce uniquely the concentrations of H^+, He^+, and O^+. The self-consistency between the data presented in this paper and those of Alouette 1, and the agreement of these data with the more direct ion mass spectrometer measurements of *Taylor et al.* [1963], together with the critical frequency ion concentration expressions, illustrate that VLF radio techniques are a useful and important method for determining ion compositions in the ionosphere.

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