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Dreicer order ambipolar electric fields at Parker's steady state solar wind sonic critical point

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Abstract. The time-independent ambipolar electric field $E(r_*)$ at Parker's solar wind sonic critical point r_* is analytically shown to range between $(0.6–2.0)$ $E_D(r_*)$, where $E_D(r_*)$ is the local *Dreicer* [1959, 1960] electric field. The ratio of ambipolar to Dreicer field strength scales as $(\langle T_* \rangle/10^{6o}\ K)^{3/2}$ $20/\ln \Lambda$, where $\langle T_* \rangle$ is the average of electron and ion temperatures at the critical point. As a million degree corona is nearly certainly required for the wind as we observe it, $E(r_*) \simeq E_D(r_*)$. Since the steady state solar wind is characterized by no parallel currents, these large electric fields require generalizations of Dreicer's discussion of what are the results of such large electric fields. This is clear since even when $E = 0.43E_D$ in a homogeneous plasma, the electron fluid drifts at the electron thermal speed with respect to the ions in the absence of collective instabilities. Consequences of such a large current at the critical point are not found at 1 AU. The electric fields at the orbit of earth [*Scudder*, 1995b] and at the base of the fully ionized layer of the transition region [*Scudder*, 1995a] are nevertheless comparable to the local Dreicer electric field. From a theoretical point of view, the finding that such large electric fields are required in the plasma points to the need for basic modifications to the macroscopic description of a magnetized plasma that are outside of the Chapman-Enskog-Spitzer-Braginskii closure schemes that assume perturbative corrections to homogeneous solutions as an expansion in the small parameter $\varepsilon \equiv E_{\|}/E_D$. It is suggested that the possibility to form gradients, to allow heat to flow, and to allow the bulk plasma to move can short circuit the homogeneous expectations of bulk runaway implicit in the Dreicer discussion. However, the intrinsic bifurcation of the electron distribution reflects the physics of the Coulomb cross section in the presence of the very strong dc field and is suggested as the underlying reason for the omnipresent nonthermal distributions inside and out of the critical point. The ambipolar electric field at the solar wind critical point from two-fluid theory and the electric field assumed in Spitzer-Braginskii heat transport are shown to disagree at almost all the possible two-fluid critical points. This same closure flaw also afflicts the fluid portion of any turbulence description that may be thought relevant for the wind.

1. Introduction

The description of the macroscopic (fluid) equations with gradients for inhomogeneous plasmas has developed in parallel with the moment descriptions of gas and magnetohydrodynamics [*Chapman and Cowling*, 1970; *Braginskii*, 1965; *Trubnikov*; 1965]. The transport description for a plasma, however, requires additional care especially in the case of the fully ionized gases of astrophysics, so that gradients do not allow unwanted space charges to develop with the attendant requirements to retain higher frequencies and shorter scales in the description of the medium. In the steady state, weak gradient limit, parallel electric fields are chosen in these transport procedures so as to preclude these effects, yielding hydrodynamic looking transport relations like $\mathbf{Q} = -\kappa \nabla T$ that permit closure of the infinite ladder of formal moments of the kinetic equation. Even attempts to include strong gradients by *Campbell* [1984] explicitly presuppose the ancillary condition that these required electric fields are small compared to the Dreicer electric field discussed below. However, the transcription and application of these closure techniques for astrophysical plasmas where gravity plays an important role poses new problems not solved by these prior approaches. Theory for a spatially inhomogeneous plasma has been developed for "weak departures" from spatial homogeneity, where in lowest order the quasi-neutrality electric field vanishes and is perturbatively adjusted to enforce electrodynamic attributes such as no parallel currents. In this way, initial quasi-neutrality is preserved in the presence of gradients by choosing $E_{\|}$ so that $j_{\|} = 0$ ensuring that charge density does not grow.

In this way approximate, local, partial differential equations have been formulated with these closure coefficients whose solutions are attempts at the macroscopic moment description of the physical system within the very fragile assumptions that allowed the system of partial differential equation (PDEs) to be ascertained (cf. *Scudder* [1992a] for a discussion of these). This type of truncated moment equations are the starting point for various inferences that the observed solar wind "needs" mechanisms for energy deposition beyond thermal replenishment. Such conclusions have been made using Spitzer-Braginskii formulas or their saturated limits. The conclusions of these arguments are only as strong as the integrity of the equations used. Similar closure flaws occur in all "tests" to date

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of the adequacy off turbulence mediated winds. In this paper the fluid's side of these equations are shown to have serious flaws that invalidate the closure schemes upon which they are built. The earlier arguments about the insufficiency of the random energy support for the solar wind expansion must be reopened, and the necessity for turbulence is no longer obvious.

The usual fluid moment equations are derived ignoring the well known bias that gravity can introduce to the microscopic description of the kinetic physics. Gravity by itself requires a parallel electric field to enforce quasi-neutrality that need not be the same electric field presupposed in the Spitzer-Braginskii transport formulation. Welding Braginskii-Spitzer transport formulae into macroscopic equations for inhomogeneous equilibria does not assure the plasma described is quasi-neutral.

A particularly well known example for this electric field is the gravitationally bound isothermal plasma atmosphere. In order that electrons and ions be distributed inhomogeneously in the gravitational field with the same scale height and pressure profile and be quasi-neutral, a radial electric field of approximately half the strength of the gravitational field is required [*Pannekoek*, 1922; *Rosseland*, 1924] to compensate for the radically different gravitational attractions of electrons and ions. (For a neutral atmosphere, species of different mass could have different scale heights with impunity.) When an ion atmosphere's dynamics is to be modeled with sources and sinks for internal energy, the weak gradient moment PDEs, together with the body gravitational force are used to relate density, momentum, and energy. These equations have the "impedance mismatch" that the transport microphysical recipes (e.g., $\mathbf{Q} = -\kappa\nabla T$) contain a choice for $E_\parallel$ that counteracts the tendency for gradients to spawn current growth without any consideration for the overarching need for the macroscopic plasma's need for quasi-neutrality.

It is often, but incorrectly, assumed that gravitationally structured plasmas can be completely addressed by adding a body force to the weak gradient moment equations and using a common continuity equation together with the weak gradient closure recipes. The structure of the conservation equations is exact if no terms are omitted. The approximation involves how pressure and heat flux are related to lower-order moments; examples are polytropes or heat conduction "laws" that allow the system of moment equations to be mathematically closed, yet with two separate $E_\parallel$ values: (1) the exact one by the difference of the two momentum equations in terms of other moments and (2) the approximate thermoelectric field based on a gradient expansion involved in the Spitzer formulation of the heat law with no currents [cf. *Shkarofsky et al.*, 1963]

$$E_\parallel^{\mathrm{Spitzer}} \simeq \frac{kT_e}{e}\hat{\mathbf{b}}\cdot\nabla\ln\left(\frac{(kT_e)^{1.703}}{n_e}\right), \tag{1}$$

where $(\Omega_{ce}/\nu_{ei} \gg 1)$ has been assumed. In the limit of the isothermal, exponential atmosphere, this expression simplifies to

$$E_\parallel^{\mathrm{Spitzer}} = -\frac{kT_{eo}}{e}\hat{\mathbf{b}}\cdot\nabla(\ln n). \tag{2}$$

For the isothermal exponential atmosphere

$$n_e = n_o \exp\left(-\frac{GM_\odot(M+m)(r-r_o)}{2kT_{eo}rr_o}\right).$$

For this special example the results of (1) and (2) above give the same, consistent electric field:

$$E_{\mathrm{iso}} = \frac{1}{e}\hat{\mathbf{b}}\cdot\nabla[\Phi_G(r)/2]. \tag{3}$$

However, if isothermal conditions cannot be met, there are still stellar atmospheres which are not isothermal with E given by (3) [cf. *Scudder*, 1992a] which disagree with (1). In this circumstance the transport and quasi-neutrality are inconsistently addressed. For example, under velocity filtration [*Scudder*, 1992a, b],

$$T(r) = T_oG(\Psi_{\mathrm{eff}}). \tag{4}$$

where

$$G(\Psi_{\mathrm{eff}}) = \left[1 + \left(\frac{2}{2\kappa-3}\right)\Psi_{\mathrm{eff}}(r)kT_o\right]. \tag{5}$$

The density varies like

$$n(r) = n_oG[\Psi_{\mathrm{eff}}(r)]^{(-\kappa+1/2)}. \tag{6}$$

In this regime the Spitzer-Braginskii gradient expansion recipe assigns $E_\parallel$ to be determined by

$$E_\kappa \simeq \frac{1}{e}\frac{2\kappa+2.406}{2\kappa-3}G^{1/2+\kappa}\hat{\mathbf{b}}\cdot\nabla(\Phi_G(r)/2). \tag{7}$$

Only in the Maxwellian limit when $G \rightarrow 1$, $\kappa \rightarrow \infty$ are (3) and (7) equal.

Only through the conservation equations can E be determined exactly and even then only when the pressure gradients and thermal force expressions are known to have an independent precision. The apparent freedom for choice of closure is illusory, since almost all choices will be inconsistent with the requirements for quasi-neutrality. Whether the choice is polytrope, incompressible flow or heat law, it is almost certainly inconsistent with the electric field necessary for quasi-neutrality.

2. Present Work

In this paper, conservation equations are used to analytically calculate the size of the ambipolar electric field necessary to maintain quasi-neutrality at Parker's sonic critical point of the solar wind expansion. This ambipolar electric field is shown to be of order unity in units of the *Dreicer* [1959, 1960] electric field E_D, which is the appropriate yardstick for a large parallel electric field. The Dreicer electric field is defined to be of a magnitude so that an electron of kinetic energy $\frac{3}{2}\mathrm{kT_e}$ gains kT_e in one mean free path in this electric field, namely,

$$eE_D\lambda \equiv kT_e. \tag{8}$$

In a homogeneous Maxwellian plasma, an electric field with $E \equiv E_{mf} = 0.43E_D$ causes a current to flow with a drift speed relative to the ions equal to the electron thermal speed [*Dreicer*, 1960]. For any higher field strength in this model, no steady state is possible between friction and the applied emf. When $E < E_{mf}$, a steady state current flows and a nonlinear Ohmic relationship obtains

$$j = \sigma(E)E \qquad E < E_D. \tag{9}$$

Ordinarily steady dc electric fields comparable to the Dreicer field have not been considered theoretically, since the Dreicer induced drifts in a homogeneous medium are unstable to microinstabilities which are widely thought to be quenched by a relaxation into a nearby state without such large electric fields. The runaway problem for finite but still small electric field regime $0 < \varepsilon < 0.05$ has been addressed in the literature [e.g., *Dreicer*, 1959, 1960; *Kruskal and Bernstein*, 1964; *Fuchs et al.*, 1986; *Moghaddam-Taaheri and Vlahos*, 1987; *Holman*, 1985, 1995] for spatially homogeneous plasmas. In all cases, transient behavior was the focus; steady states were not considered.

The demonstration contained in this paper of the steady state size of the parallel dc electric fields requires a global discussion of the kinetic behavior of the plasma, a situation that is the antithesis of the weak gradient fluid PDEs for the moments of the distribution function. It is suggested that this, until now, unrecognized situation underlies the enormous difficulties in quantitative energy budgets that plague the solar wind expansion in particular and astrophysics more generally.

3. Size of $E_{\|}$ at Parker's Critical Point

Recently, it has been analytically demonstrated [*Scudder*, 1996] (hereinafter referred to as Paper I) that Parker's necessary and sufficient conditions for the formation of a saddle x-type critical point in a spherically symmetric solar wind are corollaries of requiring that the ion's equivalent potential (the sum of the ion's gravitational and electrical potential), Ψ_+, be a local maximum at the critical point r_*. From this condition it follows that Parker's sonic critical point is the location where there is a balance of forces on an ion between the electric and gravitational fields. This implies that

$$eE(r_*) = \frac{GM_S M}{r_*^2}. \tag{10}$$

The general location for the critical point from (paper I, equation 37a) is given by

$$r_* = \frac{GM_S(M + m)}{2k\langle T_*\rangle[2 - (\beta_e\Delta_e + \beta_i\Delta_i)]}, \tag{11a}$$

where Δ_k is the partial pressure of species k at the critical point:

$$\Delta_k = \frac{T_{k*}}{T_{e,*} + T_{i,*}}, \tag{11b}$$

and where

$$\Delta_e + \Delta_i \equiv 1, \tag{11c}$$

with T_{e*} and T_{i*} the electron and ion temperatures at the critical point, respectively. The average temperature at the critical point is $\langle T_*\rangle$. The radial power law exponents of the electron and ion temperatures at the critical point are $\beta_{e,i}$.

Equation (10) can be rewritten by using one power of r_* to the left-hand side and (11a) for the other to obtain

$$eE(r_*)r_* = \frac{M}{M + m} k\langle T_*\rangle[4 - 2(\beta_e\Delta_e + \beta_i\Delta_i)], \tag{12}$$

an expression independent of explicit reference to gravity. Let the shortest scale length for a macroscopic fluid variable M be given by L_M, then

$$L_M^{-1} = \frac{d \ln M}{dr} \equiv \frac{|\zeta|}{r_*}. \tag{13}$$

Defining the Knudsen number for momentum transfer for electrons, K_n, at r_* by

$$K_n(r_*) \equiv \frac{\lambda_{\mathrm{mom}}}{L_M} = \frac{|\zeta|\lambda_{\mathrm{mom}}(r_*)}{r_*}, \tag{14}$$

where λ_{mom} is the mean free path for momentum transfer of the RMS speed particle, (10) can be rewritten as

$$eE(r_*)\lambda_{\mathrm{mom}} = kT_{e*}K_n \frac{M}{M + m} \frac{(2 - (\beta_e\Delta_e + \beta_i\Delta_i))}{\Delta_e|\zeta|}. \tag{15}$$

The strongest macroscopic variation $|\zeta|$ for almost any fluid variable at the critical point is (via paper I) that of the density. In this approximation, using (22b) and (38b) in paper I, we obtain

$$|\zeta| = |\delta| = \frac{2 - \Delta_i\beta_i}{\Delta_e} \tag{16}$$

Ignoring m/M compared to unity and using (16) into (15) yields

$$eE(r_*)\lambda_{mfp} = kT_{e*}K_n\left(1 - \frac{\beta_e\Delta_e}{2 - \Delta_i\beta_i}\right) \cdot (|\zeta| \simeq -\delta) \tag{17}$$

The Dreicer electric field E_D is that applied emf for which there is no longer enough coulomb friction from the entire electron fluid, for there to be an equilibrium between coulomb drag and the emf, provided f(v) remains close to a Maxwellian and the system is spatially homogeneous [*Dreicer*, 1959, 1960]. The size of this electric field is given by (8). The existence of such a limit is usually demonstrated by showing that the friction provided by the Coulomb interaction has a limited Stokes regime where resistance is proportional to the first power of the relative drift speed. At larger drifts this friction increases more slowly with drift speed than the first power, until at a drift speed of the thermal speed of the electrons the friction stops growing altogether and thereafter decreases with increasing drift speed. The maximum friction electric field occurs when

$$E_{mf} = 0.43E_D. \tag{18}$$

When the applied emf exceeds E_{mf} the drift speed of electron center of mass grows without bound in a homogeneous plasma with postulated drifting Maxwellian distribution function. Such large currents are often not replenishable, nor is there sufficient energy for this to represent a steady state. Long before the order one regime relative to Dreicer is achieved, large drift velocities, and hence, currents are implied even when a homogeneous sub-Dreicer equilibrium is mathematically possible. The Dreicer limit was examined for laboratory machines to ascertain what emf's would cause the electrons to "runaway" to the container walls, leaving the ions behind and causing a total loss of confinement. These findings have often led to the extrapolation that parallel electric fields of this size in nature could only precipitate transient behavior, rather than be part of a steady state equilibrium. Apparently unconsidered was the possibility that large electric fields could be present, counteracted by pressure gradients, but with no dc current flowing.

Nevertheless, comparison of (17) and (8) reveals that

$$\varepsilon \equiv \frac{E(r_*)}{E_D} = 2.3\left(1 - \frac{\beta_e\Delta_e}{2 - \Delta_i\beta_i}\right) K_n, \qquad (|\zeta| \simeq -\delta) \tag{19}$$

a result that demonstrates the order unity size of ε at the critical point. This is clear since the Knudsen number is also

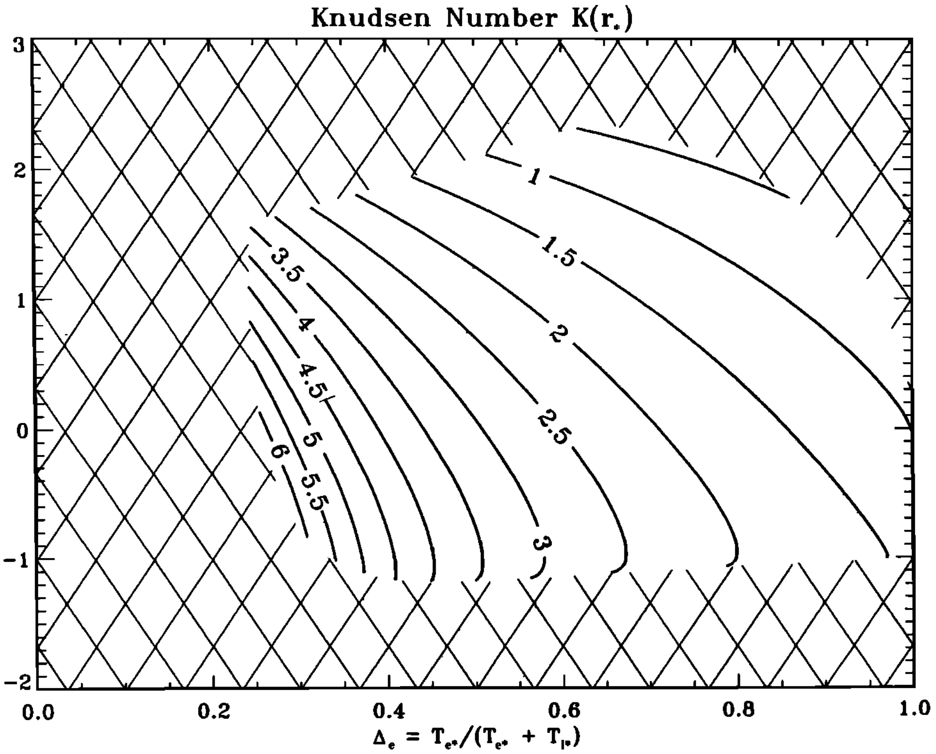

Figure 1. Knudsen number $K(\Delta_e, \beta_e)$. Cross-hatched region is excluded portion of parameter space where no critical points are found.

near unity at the critical point, a place often used as the base conditions for exospheric treatments of the solar wind [e.g., *Lemaire and Scherer*, 1971] (cf. Figure 1).

The Knudsen number at the critical point can be computed so that E/E_D of (19) may be explicitly and self-consistently evaluated. With the additional knowledge of wind speed at the critical point in terms of the critical point temperature:

$$U(r_*) = \left(\frac{2k\langle T_*\rangle}{m+M}\right)^{1/2} \tag{20a}$$

and the observed mass flux at 1 AU of the solar wind, together with conservation of mass

$$n(r_*)U(r_*)r_*^2 \equiv n_*U_*r_*^2 = n_{1\,\mathrm{AU}}U_{1\,\mathrm{AU}}r_{1\,\mathrm{AU}}^2 \equiv n_oU_or_o^2, \tag{20b}$$

the density at the critical point is determined as

$$n_* = \frac{n_oU_or_o^2}{U_*r_*^2}. \tag{20c}$$

The free path for the root-mean-square speed particle is [*Rossi and Olbert*, 1970]

$$\lambda_{\mathrm{isotropization}}(r_*) = \frac{8.8\times 10^4 T_e^2(r_*, {}^\circ K)}{n(r_*, cc^{-1})\ln\Lambda(r_*)}\ \mathrm{cm}. \tag{21}$$

As illustrated in Figure 1, the Knudsen number via (14) and (21) is thus determined across the possible critical points. As presupposed previously, but as now actually calculated for the first time, the Knudsen number at the critical point exceeds unity for almost all choices, ranging from 0.5 to 6.0. With this result the Dreicer scaled electric field at the critical point can be found after using $T_{e*} = 2\Delta_e\langle T_*\rangle$ to be

$$\frac{E(r_*)}{E_D(r_*)} = \left(\frac{M}{M+m}\right)\left(\frac{4-2(\beta_e\Delta_e+\beta_i\Delta_i)}{\Delta_e}\right) \cdot \frac{3.6\times 10^5\Delta_e^2\langle T_*\rangle^2U_*r_{*,\mathrm{iso}}}{n_oU_or_o^2\ln\Lambda}\left(\frac{r_*}{r_{*,\mathrm{iso}}}\right). \tag{22}$$

From (paper I, equation (2c))

$$r_{*,\mathrm{iso}} = \frac{GM_S(M+m)}{4k\langle T_*\rangle} \tag{23}$$

together with (20a) yields

$$U_*r_{*,\mathrm{iso}} = \frac{GM_S}{2}\left(\frac{2k\langle T_*\rangle}{m+M}\right)^{-1/2}. \tag{24}$$

This gives the dimensionless electric field of (22) in the form

$$\frac{E(r_*)}{E_D(r_*)} = \left(\frac{M}{M+m}\right)\left(\frac{4-2(\beta_e\Delta_e+\beta_i\Delta_i)}{\Delta_e}\right) \cdot \frac{3.6\times 10^5\Delta_e^2\langle T_*\rangle^2U_*r_{*,\mathrm{iso}}}{n_oU_or_o^2\ln\Lambda}\left(\frac{r_*}{r_{*,\mathrm{iso}}}\right). \tag{25a}$$

Specializing (25a) to the conditions of our wind at 1 AU of $n_o = 8/cc$, and $U_o = 4.0\times 10^7$ cm/s (notice that n_oU_o is relatively constant at 1AU, cf. *Bridge* [1976]), the final dimensionless electric field at the sonic critical point is obtained:

$$\left.\frac{E(r_*)}{E_D(r_*)}\right|_{\mathrm{sun}} = 2.48\Delta_e\left(\frac{\langle T_*\rangle}{10^{6o}K}\right)^{3/2}\left(\frac{20}{\ln\Lambda}\right)\left(\frac{8/cc}{n_o}\right) \cdot\left(\frac{400\ \mathrm{km/s}}{U_o}\right). \tag{25b}$$

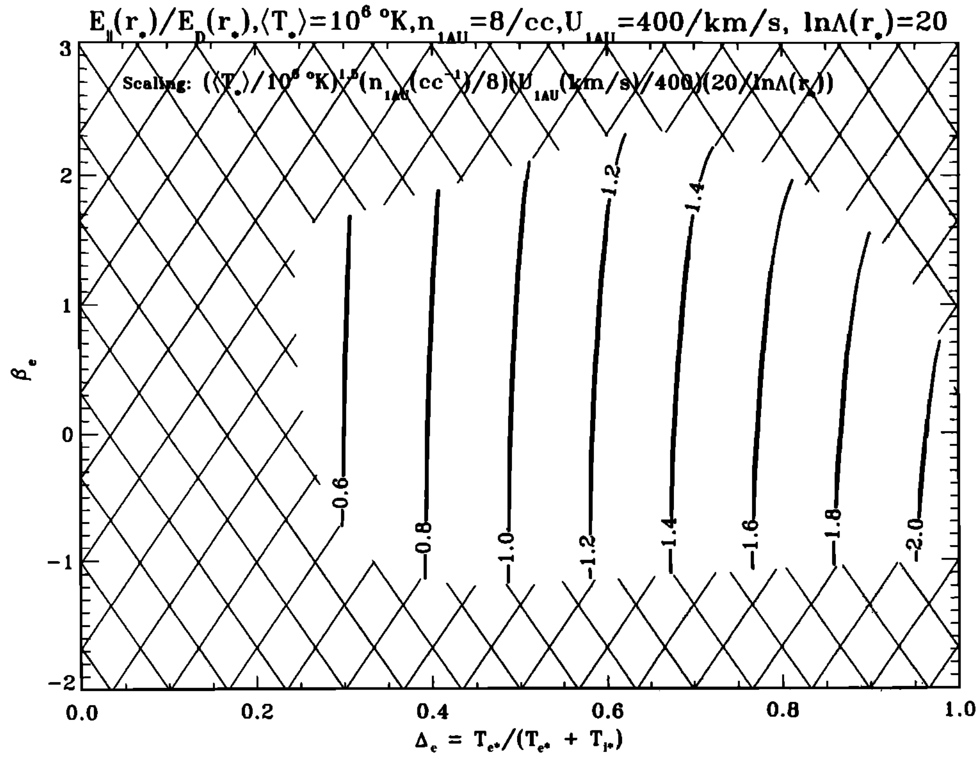

Figure 2. $E(r_*)/[E_D(r_*)]$ as a function of allowed Δ_e, β_e where critical points occur for two-fluid wind. Cross-hatched region is excluded portion of parameter space where no critical points are found.

Using the allowed critical points from (paper I) and assuming $\langle T_* \rangle = 10^6$ °K, Figure 2 illustrates the variation and numerical size of the dimensionless electric field at the possible critical points. Only the Coulomb logarithm's variability at the different critical points of this diagram causes the weak dependence of ε on the electron temperature power law exponent at the critical point, β_e. For equal electron and ion temperatures, $\Delta_e = 0.5$, the dimensionless electric field is essentially unity, and almost strictly proportional to the partial pressure fraction of the electrons at the critical point.

From Figure 2 and the form of expression (25b) it is clear that the formation of a solar wind with the observed 1 AU mass flux that requires million degree temperatures at the critical point is synonymous with large, Dreicer order, electric fields at the critical point. The velocity space local runaway production in this locale is thus an unavoidable corollary of the steady state of the solar wind.

4. $E_\parallel \simeq E_D$ and $J_\parallel = 0$?

In steady state

$$\frac{j_{\parallel,1}}{B_1} = \frac{j_{\parallel,*}}{B_*} \tag{26a}$$

so that any thermal speed scaled relative drift at the critical point would translate into a relative slippage at Earth of

$$\frac{v_d(1\text{ AU})}{v_{th}(1\text{ AU})} = \left(\frac{B(1\text{ AU})n(r_*)}{B(r_*)n(1\text{ AU})}\right)\left(\frac{T(r_*)}{T(1\text{ AU})}\right)^{1/2}$$

$$\simeq \left(\frac{U(r_*)}{U(1\text{ AU})}\right)\left(\frac{T(r_*)}{T(1\text{ AU})}\right)^{1/2} \simeq 0.8, \tag{26b}$$

where use has been made of the ratio of bulk speeds between the critical point and 1 AU is approximately 1/4. Observations [e.g., *Montgomery et al.*, 1968; *Scudder et al.*, 1986] show that the electron drift speed in units of the thermal speed is experimentally small, consistent with being zero with a precision of 25 parts in 1800,

$$\frac{v_d(1\text{ AU})}{v_{th}(1\text{ AU})} = 0 \pm 0.014, \tag{27}$$

contradicting the premise of (26b). For a steady state solar wind, the coherent flow of $J_\parallel$ from the corona on open solar wind field lines is inconsistent with charge neutrality of the Sun. Further, steady parallel currents would also reorganize the scale of the directional changes in the interplanetary magnetic field so that the Parker spiral would not be detectable. Therefore the consequences of such large $E_\parallel$ values are not found in heretofore undetected current flows.

5. $E_\parallel \simeq E_D$ and $f(\mathrm{v})$ a Gaussian?

When studying the role of the electric field on the electron distribution function, *Dreicer* [1959] and *Fuchs et al.* [1986] have shown that the size of this electric field naturally partitions the velocity space into a regime where the characteristics are hyperbolic and a low-energy regime where the behavior is more nearly parabolic. This separatrix occurs at a speed v^* given by

$$v^* = w_{T,e}\left(\frac{2E_D}{E}\right)^{1/2}, \tag{28}$$

where $w_{T,e} = \sqrt{(3kT_e/m)}$ is the root-mean-square speed of the distributions. For electrons above this speed the electric field force is underdamped; below it the electrons are overdamped by collisions.

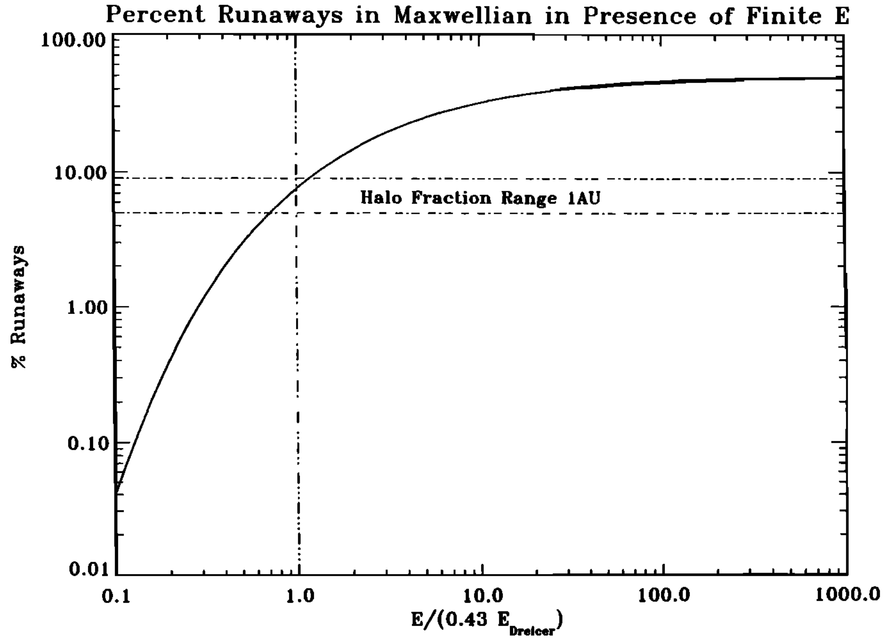

Figure 3. Fraction of Gaussian population subject to runaway as a function of the ambient electric field. In the extremely weak electric field regimes, the fraction susceptible to runaway becomes very small. Horizontal dashed lines indicate regimes of halo number fraction from long-term 1 AU averages [*Feldman et al.*, 1975].

In the homogeneous $E = 0$ limit this speed boundary, v^*, receeds to very large values where there are essentially no ambient electrons to be promoted into runaway. For a finite electric field strength the energy boundary above which particles are accelerated into runaway defines a separatrix between thermal and suprathermal electrons at the finite kinetic energy given by

$$E^* = 3kT_e(E_D/E). \tag{29}$$

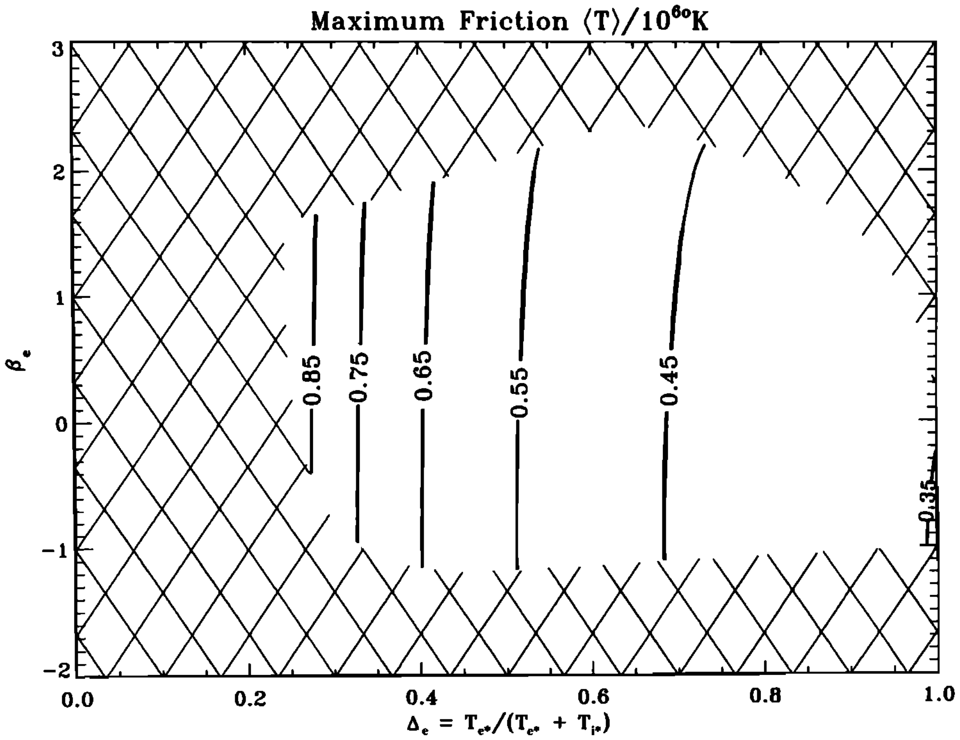

Figure 4. Average of electron and ion temperature, $\langle T_*(\Delta_e, \beta_e)\rangle$, at the critical point such that the electric field at the critical point is the maximal friction electric field. Cross-hatched region is excluded portion of parameter space where no critical points are found.

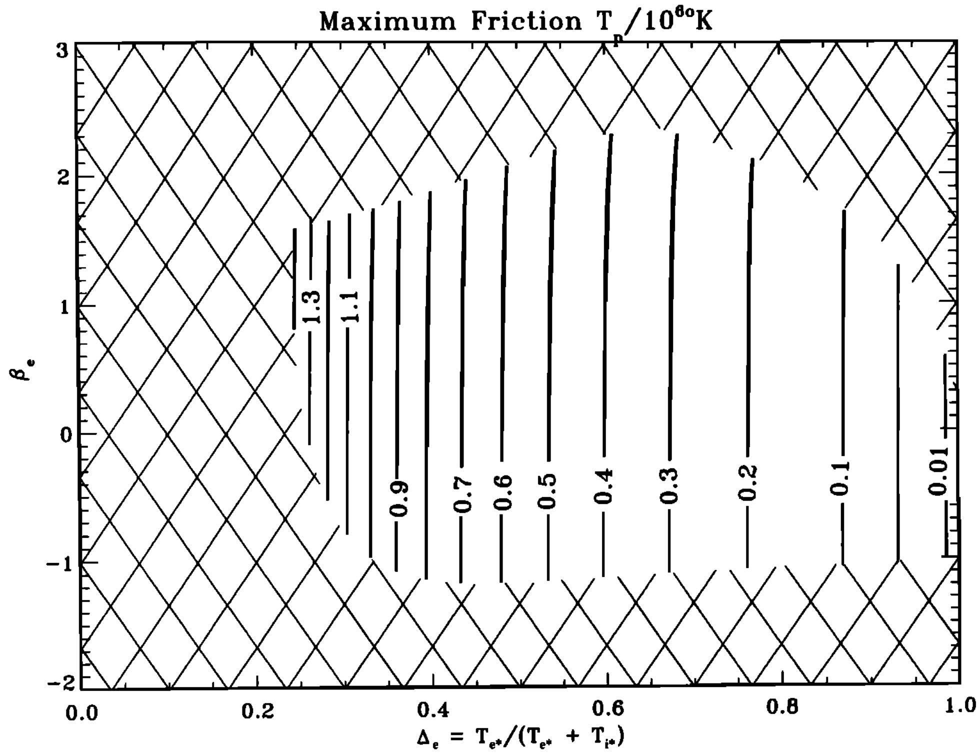

Figure 5. Ion temperature at the critical point, $T_p(\Delta_e, \beta_e)$, such that the electric field at the critical point is the maximal friction electric field. Cross-hatched region is excluded portion of parameter space where no critical points are found.

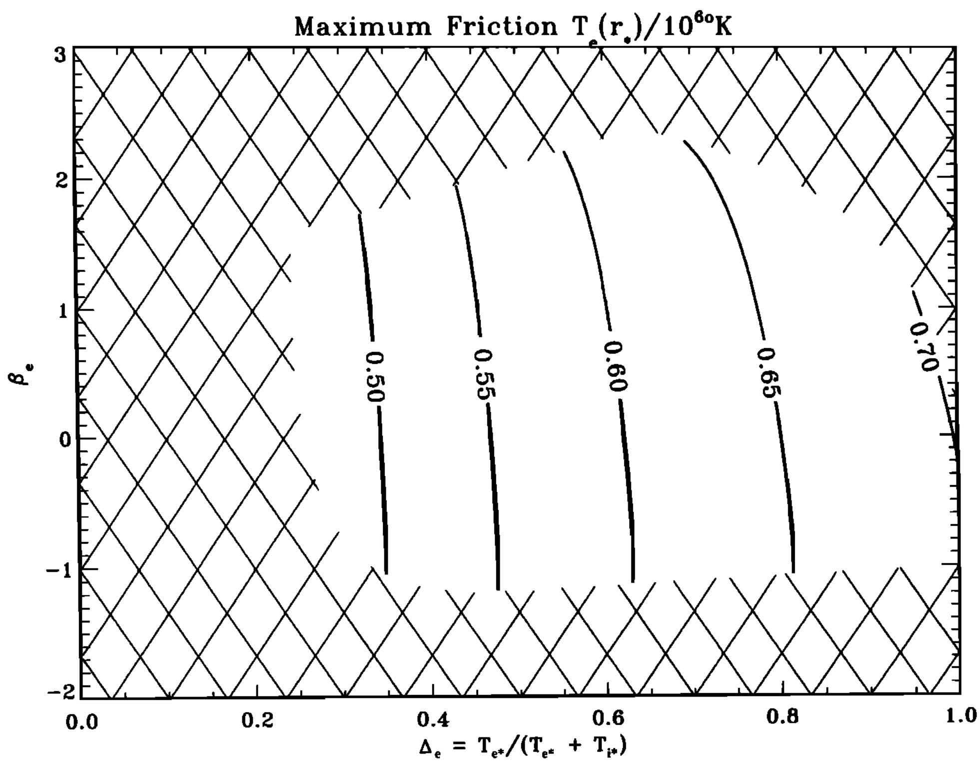

Figure 6. Average of electron and ion temperature, $T_e(\Delta_e, \beta_e)$, at the critical point such that the electric field at the critical point is the maximal friction electric field. Cross-hatched region is excluded portion of parameter space where no critical points are found.

The fraction of particles of a Gaussian affected in this way is approximately half of the particles outside the speed v^*:

$$\frac{n_{\text{run}}}{n_{\text{total}}} \simeq \frac{1}{2}\left(1 - \frac{2}{\sqrt{\pi}}\,\gamma\left(\frac{3}{2}, \sqrt{\frac{3E_D}{E}}\right)\right), \tag{30}$$

illustrated in Figure 3. In contrast to the astrophysical situation, the postulated infintesimally small electric fields of Spitzer-Braginskii closure approximation ensures the fraction of the electron gas susceptible to runaway is vanishingly small. Under the maximum friction hypothesis with $E_{mf} = 0.43E_D$ the electron distribution functions at the critical point should be bifurcated above, but in the vicinity of $7kT_{e,*}$. Above this energy about 7% of the ambient density should be found as illustrated in Figure 3. (It should be noted that this is the same regime where the suprathermal electrons are noticeably removed from a local Maxwellian at the orbit of Earth [*Montgomery et al.*, 1968; *Feldman et al.*, 1975; *Scudder and Olbert*, 1979], and, where long-term averages of this number fraction are found to be 7 ± 2% [*Feldman et al.*, 1975]).

To appreciate the importance of this finding it should be noticed that the typical regimes of "runaway" examined by fusion codes [e.g., *Fuchs et al.*, 1986] are regimes where $E/E_D < 0.04$, making the runaway population (0.03% by number) above $E^* > \simeq 75kT_e$, a regime where they are still test particles, where they are still insignificant carriers of energy or momentum in the transport. The situation suggested in this paper for the solar wind expansion at the critical point strongly suggests that nonthermal electron distributions are required as part of the equilibrium state. Finally this nonthermal part of the velocity space is now required by steady state considerations and not in the extreme cosmic ray regime of the random distribution as it has usually been modeled, but in the traditional area where suprathermal electrons are routinely found by in situ particle detectors [*Montgomery et al.*, 1968; *Ogilvie et al.*, 1971; *Feldman et al.*, 1975; *Ogilvie and Scudder*, 1978].

The "runaway" separatrix in velocity space in the solar wind electron velocity distribution at Parker's sonic point occurs in the transthermal portion of the velocity distribution [*Scudder and Olbert*, 1979], the boundary between thermal and suprathermal. This has the consequence of a steady state local runaway of a significant fraction of the gas, a situation that cannot be ignored or handled by a (1) local or (2) perturbative statistical description. This newly discovered parameter regime at the critical point is decidedly inconsistent with transport physics predicated on perturbations to nearly Maxwell-Boltzmann distribution functions [cf. *Scudder*, 1992a]. This finding reopens the question of whether internal energy rearrangements can fuel the solar wind type expansions since the large steady state electric fields calculated here invalidate the premises of Braginskii style transport used to estimate the size, the direction, or the sufficiency of the conduction flux as for example summarized by *Holzer and Leer* [1980].

6. T_{e*} and T_{p*} If $E_{\|} = E_{mf}$

Dreicer argues that there is a maximum applied electric field E_{mf} beyond which the phase space overlap of electrons and ion velocity distributions can no longer provide sufficient momentum transfer to balance the applied emf. In his homogeneous system with the proviso that electrons remain Maxwellian while drifting in response to the applied emf, he ascertained that this "maximal friction" electric field occurred when

$$E_{mf} \simeq 0.43E_D. \tag{31}$$

The existence of such a maximum E consistent with equilibrium follows from the properties of the Coulomb cross section. The exact size of this threshold depends on the precise form of the equilibrium distribution. If the equilibrium distribution does not depart substantially from the Gaussian at low energy, the maximal friction estimate may not be too far off. This is qualitatively clear since it is the low-energy electrons interacting with the protons that provides almost all the friction between the species.

If we assume (31) sets this limit, interesting inferences for the limiting values of the average electron and ion temperatures and their ratios at the critical point may be determined as parameterized by the critical points possible in the (Δ_e, β_e) space that spans all critical points. As shown in Paper I, no critical points are possible for an electron partial pressure ratio $\Delta_e < \Delta_* = 0.2437$. Equation (25b) may be set equal to 0.43 and solved for $\langle T_* \rangle$, parametric in the choice of $\Delta_e > 0.2437$. With this knowledge the electron and ion temperatures at the critical point have been illustrated in Figures 4, 5, and 6 using $T_e(r_*) = 2\Delta_e\langle T_* \rangle$ and $T_p(r_*) = 2(1 - \Delta_e)\langle T_* \rangle$. The average temperature $\langle T_* \rangle$ at the critical point illustrated in Figure 4 ranges from $\sim 10^4$ °K in the unlikely regime, ($\Delta_e \rightarrow 1$), where the ion temperature T_{p*}, approaches zero (cf. Figure 5) at the critical point, to 9×10^5 °K for the lowest allowable electron partial pressure Δ_e. In the highest average temperature regime the ion temperature T_{p*} would be in excess of 10^6 °K (Figure 5), near the canonical values. The electron temperature at the critical points surveyed vary over a much narrower range illustrated in Figure 6 (4.5×10^5 – 7.0×10^5 °K), in anticorrelation with the ion temperature behavior. There is a maximum proton temperature ratio at any critical point of

$$\frac{T_p(r_*)}{T_e(r_*)} < 3.10 \tag{32}$$

as implied by the minimum Δ_e noted above. In this (expected) regime the ion temperature at the critical point exceeds the electron temperature.

These estimates of the electron temperature at the critical point may shed some light on the extrapolations made by *Marsch et al.* [1989] concerning the electron temperature gradients inside Helios' orbit. These authors inferred a marked change in the radial profile of electron temperature would be required to match the innermost Helios data with an assumed 2×10^6 °K common electron and ion temperature at the coronal base. Since the electrons of Figure 6 are almost always cooler than this at the critical point, the extrapolated radial gradients within 0.3 AU need not be as different from their observed behavior outside 0.3 AU. Notice from Figure 3 that if $T_e \simeq 2 \times 10^{6\circ}$ K (as assumed by Marsch et al.) that $E/E_D \simeq 2^{3/2}$ at the critical point implying that 90.5% of the electrons at the critical point would participate in the local runaway. Using the electric field estimate based on maximal friction, this local runaway density would be reduced to only 7.2% in local runaway, a number comparable to the number in the halo population at the orbit of Earth [*Feldman et al.*, 1975].

Under the maximum friction assumption, the location of the critical point via (11a) can be determined in units of solar radii. Since the parameters that characterize the critical point are all within typical parameters, it is not surprising that the critical

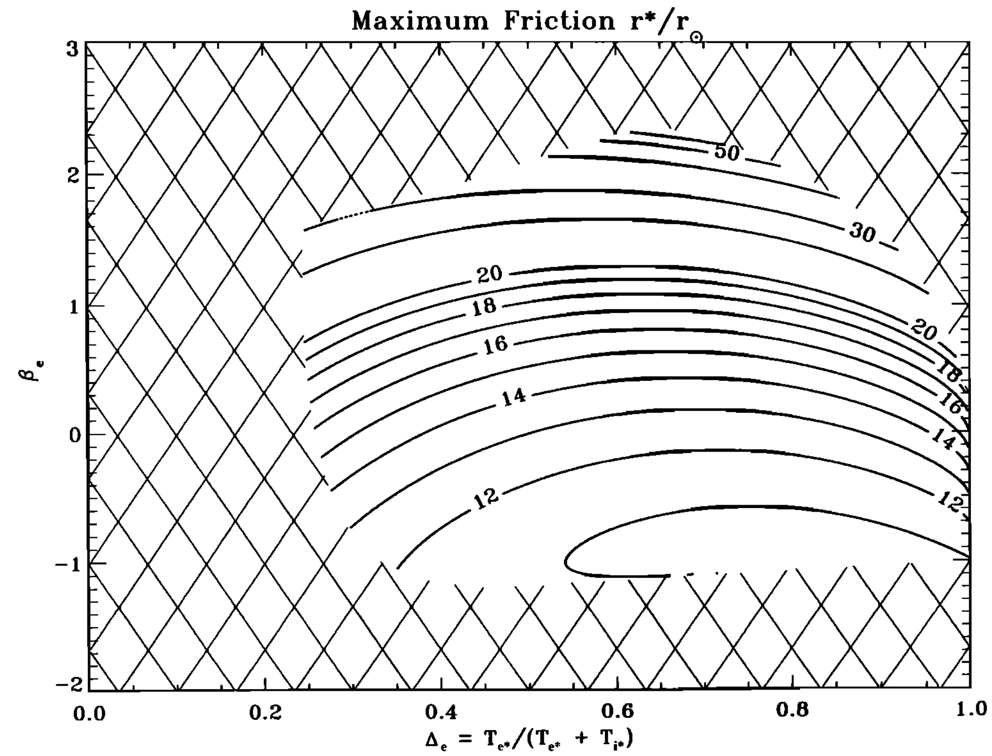

Figure 7. The location of the critical point under the assumption of the maximum friction hypothesis. Levels are at [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 25, 30, 40, 50, 60] R_S. Cross-hatched region is excluded portion of parameter space where no critical points are found.

points should be similarly reasonable as illustrated in Figure 7. There is no apparent contradiction with the temperature at the critical points inventoried here under the assumption that the electric field there is at the maximal friction value of E_D.

7. $E_{\parallel}(r_*)$ Versus E_{Spitzer}

Finally, it is now possible to make the comparison with Spitzer's transport choice of the electric field (1) and that found here as required by the steady state two-fluid equations (10). As we know, the required electron and ion temperature gradients and those of the density at the critical points, we may calculate (1) and (10) directly and exhibit their ratio independent of the maximum friction estimates of Figures 4–6 made in the previous section. It is found to be

$$\frac{E_{\mathrm{Spitzer}}}{E(r_*)} = \frac{1.703\Delta_e\beta_e + \Delta_i\beta_i - 2}{\Delta_e\beta_e + \Delta_i\beta_i - 2}, \tag{33}$$

where use has been made of (22b) and (38b) in Paper I; this expression is illustrated in Figure 8. The Spitzer electric field even has the incorrect sign for some electron partial pressures when $\beta_e > 1.7$. The quasi-neutrality electric field is stronger than Spitzer would estimate for $0 < \beta_e < 1.7$, and weaker in the traditional area of $\beta_e < 0$. Figure 8 clearly shows that quasi-neutrality and closure approximations when imposed independently can often contradict one another. The ultimate asymptotic wind speed is controlled by the net accelerating force supplied by the electric force to the ions, so the underestimate of Spitzer E even in the case of decreasing $\beta_e < 0$ profiles has underestimated the electric fields efficiency for accelerating the gas. A complimentary statement is that the heat flow permissable in that regime has also been underestimated.

8. Discussion

The large electric fields required by steady state considerations of conservation equations can be reconciled with the transient "runaway" expectations by considering three differences between astrophysical plasmas and the laboratory devices plagued by transient runaway loss of containment. (1) The parallel electric fields of astrophysics are internally forced by gravity and its induced gradients, while laboratory emf's are usually externally imposed. Independent of transport considerations, gravity requires an electric field to achieve quasi-neutrality. (2) The fixed, finite size of the containers of laboratory devices placed constraints on how such large electric fields could be accommodated: in particular, the transient electron current to the walls, rendered the remaining nonneutral plasma uninteresting as it escapes confinement. In the solar wind, for example, the large electric field has other outlets for the apparent impasse precipitated by local runaway production: without confining boundaries the volume occupied by the electrons and ions could change in such a way to rearrange the plasma density and temperature to counteract this large electric field. Momentum balance at the fluid level can be achieved for electrons not just with friction, but with opposing electron pressure gradients and cooperative electron and ion hydrodynamic bulk motion in ways not possible in homogeneous, small laboratory vessels. (3) There are other odd moments besides the respective number fluxes for the ions and electrons, such as heat flux, which can respond under such large emf's. Such

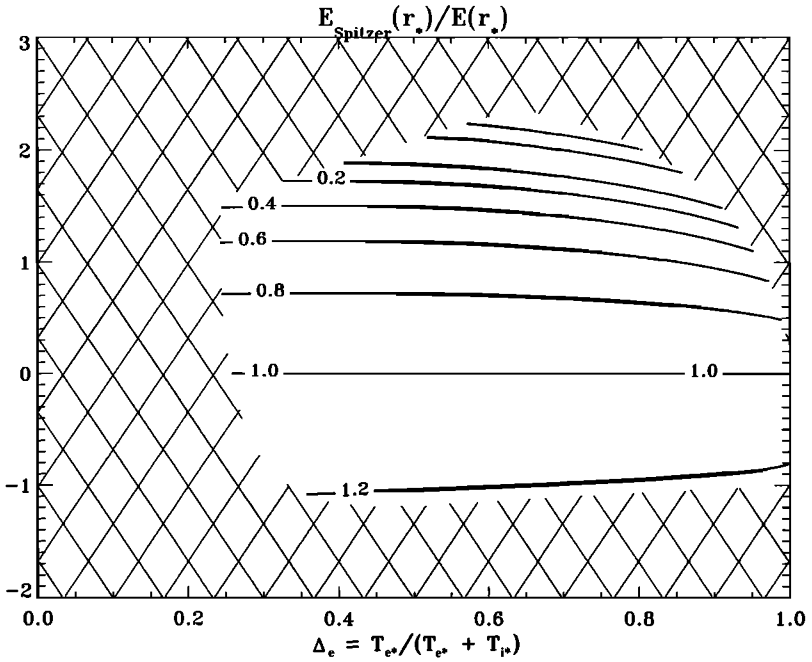

Figure 8. The ratio of the electric field implicit in the use of Spitzer-Braginski conduction formula and the electric field required by the conservation equations for the wind in the presence of gravity. Notice that this result is independent of the maximum friction estimates of Figures 4–6. Contour levels are [−0.4, −0.2, 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2]. Cross-hatched region is excluded portion of parameter space where no critical points are found.

skewed velocity distributions were not considered by Dreicer; they can also enhance the frictional coupling to the ions without drawing currents. By redefining the spatial volume occupied by the plasma, an astrophysical plasma could also develop an equilibrium steady hydrodynamic expansion in the direction of the electric field. In fact, in the exospheric theory of long mean free path gases the electric field plays the role of an extracting electric force on the ions. In the case of the laboratory this possibility of an orderly expansion was preempted by the bounding laboratory container but is clearly a force behind omnipresent stellar winds expanding into the relative vacuum of interstellar space and the cause of the nonthermal distributions as well.

By the persistence of the observed solar wind we are impelled by observations to seek a relatively steady state explanation for the wind and, with the results of the present paper, we now must do so in the presence of electric fields that are not perturbatively small (as is assumed in all fluid transport descriptions with fluid moment truncation) and with a suitably generalized discussion of thermal communication that is beyond that implied by Spitzer-Braginskii formulas.

It is proposed that the large E/E_D computed here is compatible with steady state distributions of plasma with (1) nonthermal distributions with bifurcated suprathermal populations locally in runaway and (2) spatial inhomogeneous lowest-order distributions of matter with lowest-order pressure gradients, skews, and hydrodynamics that counteract the seemingly unavoidable but unobserved, electron bulk runaway predicted on the basis of naive extrapolation of Dreicer's calculations. In this way there is velocity space "runaway" without configuration space runaway. The price for avoiding current flow and a charging sun is that the non-Maxwellian distribution must be considered as part of the equilibrium in this plasma.

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